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The luminosity-redshift relation in brane-worlds: II. Confrontation with experimental data

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Abstract

The luminosity distance – redshift relation for a wide class of generalized Randall-Sundrum type II brane-world models with Weyl fluid is compared to the presently available supernova data. We find that there is a class of spacially flat models with different amounts of matter Ω_ρ and Weyl fluid Ω_d, which have a very similar fitting quality. The best-fit models are equally likely and can be regarded as extensions of the ΛCDM model, which is also included. We examine three models with different evolutionary history of the Weyl fluid, characterized by a parameter $\alpha = 0$, 2 and 3. The first model describes a brane which had radiated energy into the bulk some time ago, but in recent times this energy exchange has ceased and only a dark radiation ($\alpha = 0$) is left. In the other two models the Weyl-fluid describes a radiating brane throughout the cosmological evolution, up to our days. We find that the throughout of the fitting surface extends over a wider Ω_d-range with increasing α, but the linear correlation of Ω_d and Ω_ρ holds all over the examined Ω_d range.

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1. Introduction

Current observational data [1-3] suggest that the cosmological model of a Universe with only baryonic matter has to be modified. In the easiest way, the model can be reconciled with observations by the introduction of a cosmological constant Λ and of considerable amount of dark matter (ΛCDM model). Because the energy densities of both baryonic and dark matter decrease during cosmological evolution, the cosmological constant will dominate the late-time evolution. This process was first suggested for the explanation of Ia supernovae data, which suggest that our Universe has reached an accelerating phase. In a Λ-dominated universe, the luminosity distance increases faster with redshift than in the model without Λ [4], exactly as required by the supernova data.

Generally, the agreement with experiments can be achieved by introducing a dark energy component of the Universe, which replaces Λ. Such a dark energy in general does not clump. A recent analysis [5] shows that a dark energy model with varying dark energy density going through a transition from an accelerating to a decelerating phase at redshift 0.45 fits well the observational data. Based on observations, the dark energy equation of state $w = p/\rho$ is within about -1 ± 0.1 [6].

It has been expected for some time that alternative gravitational theories, motivated by string/M-theory could replace dark matter and dark energy by geometric effects. The curved generalizations (see for example the review [7]) of the original Randall-Sundrum type II model [8] consist of a hypersurface with tension λ (the brane), representing our observable universe, embedded in a 5-dimensional space-time (the bulk). Gravitational dynamics on the brane is governed by an effective Einstein equation [9,10]. The sources of gravity in the effective Einstein equation include terms due to the asymmetric embedding of the brane into the bulk [10], nonstandard model fields in the bulk, and even quantum corrections approximated as induced gravity effects [12-15].

The most relevant source term for early cosmology is a quadratic source term in the energy-momentum tensor [11]. This term dominates over the linear term before the Big Bang Nucleosynthesis (BBN). In the simplest case of cosmological symmetries and suppression of the energy exchange between the brane and the bulk and whenever the bulk contains a static black hole, the Weyl curvature of the bulk generates a so-called Weyl fluid effect on the brane. In Fig 1 of our companion paper [16] (to be referred in what follows as paper I) we classify the different brane-world theories and their inter-relations. They are divided into two branches, one containing the original Randall-Sundrum type II model (BRANE1) and the other the flat DGP model (BRANE2). The model with Weyl fluid belongs to the BRANE 2 branch.

Supernova data were confronted with the induced gravity models [17-20]. When they are combined with the Sloan Digital Sky Survey (SDSS) baryonic peak, these seem to rule out the flat DGP models [17,18]. However it was argued in [20] that the Cosmic Microwave Background (CMB) shift parameter can over-turn this conclusion. Structure formation and CMB were also considered in the DGP models in Ref. [21].

Most recently, the authors of [22] tested the accelerating phase of the universe's expansion with a comparison of the models and the supernova data. They have tested the ΛCDM model, the DGP model and three wCDM models with equations of state where $w(a)$ (i) was constant with scale factor a, (ii) varied as $w(a) = w_0 + w_a(1 - a)$ for redshifts probed by the supernovae but fixed at -1 for earlier epochs, and (iii) varied as $w_0 + W_a(1 - a)$ since the recombination. Their

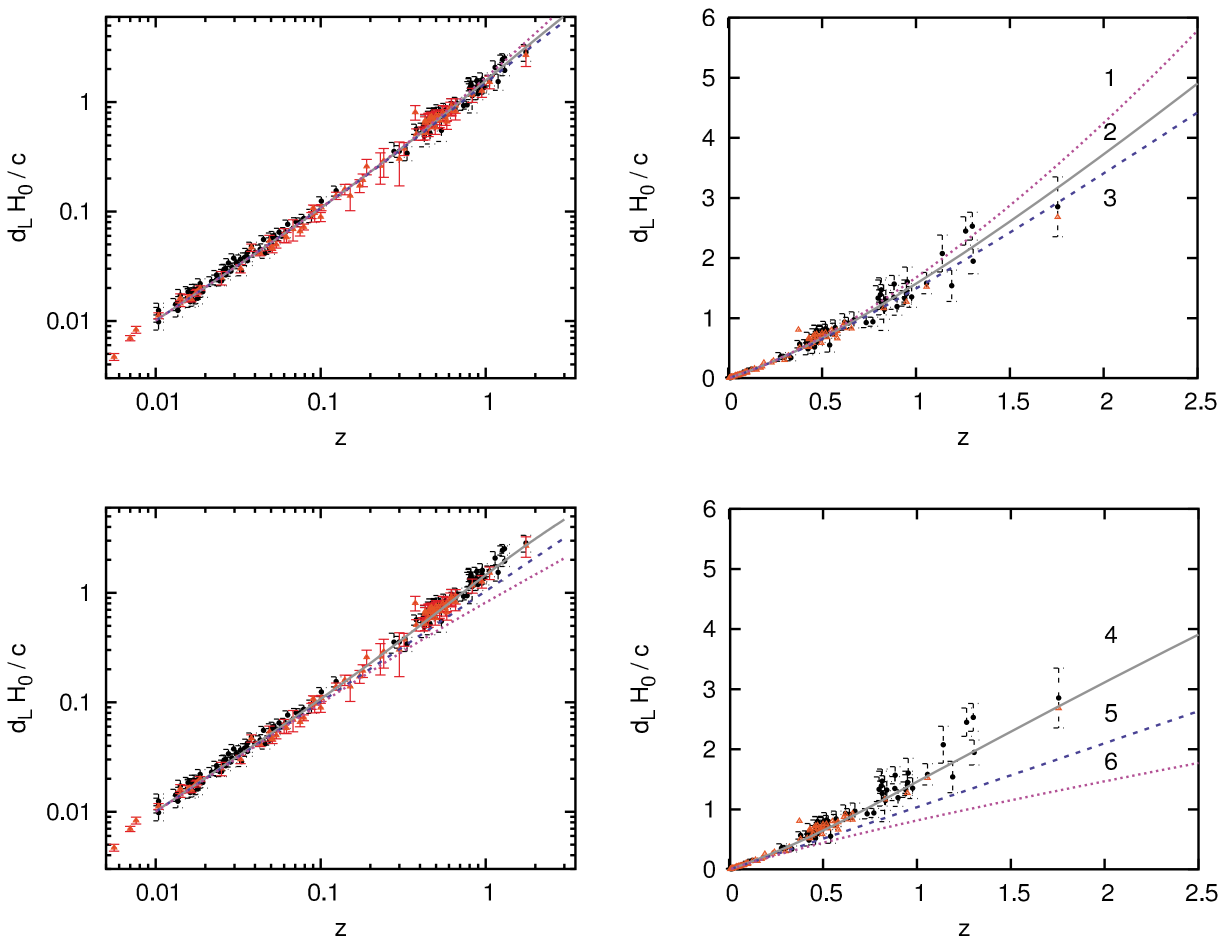

Figure 1
(Color online) Luminosity distance – redshift relations for selected brane-world cosmologies and for the ΛCDM model, compared to the supernova data. The diagrams are log-scaled (left panel) and linearly scaled (right panel). Selected low absorption supernovae from Ref. [41] are plotted with red, black dots represent the Gold set [23]. Both sets are represented with the corresponding error bars on the log-scaled diagrams. For the sake of perspicuity, the error bars of low absorption supernovae are not represented on the linearly scaled diagrams. The plotted models are the ΛCDM model (2); the brane models with cosmological constant and late-time dark radiation (1 and 3); without cosmological constant but with dark radiation (5); with cosmological constant satisfying $\Lambda = \kappa^2\lambda/2$, thus low brane tension (4 and 6) and no dark radiation.

main conclusion is that all the five examined models explain equally well the acceleration, and none of them could be selected as a preferred model, based on the Ia-type supernova data.

The authors of Ref. [18] have compared the predictions of the flat BRANE1 and BRANE2 models to the Gold [23] and Supernova Legacy Survey (SNLS) [24] supernova data sets, incorporating the baryon acoustic peaks into the analysis. These brane-world models in certain parameter range (when their induced gravity parameter Ω_l is small; the flat DGP models falling outside this range) are satisfied by both supernova data sets. The BRANE1 models fit better to the SNLS data, while the BRANE2 models fit better to the Gold data set. Since the analysis depends very weakly on the bulk cosmological constant $\tilde{\Lambda}$, the value of $\tilde{\Lambda}$ was fixed at zero. With this modification, the

BRANE1 model fits better to the SNLS data than the ΛCDM model and fits comparably well to the Gold data. The same conclusion holds for the BRANE2 model. In the analysis of [18] the dark radiation dimensionless parameter Ω_d is switched off.

Using two recent supernova data sets, the CMB shift parameter, and the baryon oscillation peaks, the authors of Ref. [25] have found that the LDGP model (a subclass of the BRANE1 models with the effective energy density having a phantom-like behavior due to extra-dimensional effects, see Fig 1 of Paper I) fits the observations if it is very close to the ΛCDM model. The modification of the LDGP model with respect to the ΛCDM model appears in the form of a linear term in the Friedmann equation, H/r_c, where H is the Hubble parameter and r_c a crossover scale. This model includes a cosmological constant, possibly screened by the modified gravity, however the comparison with observations sets strong constraints on the screening.

The first comprehensive study of the generalized Randall-Sundrum type II (RS) brane-worlds tested against astronomical data was presented in Ref. [26]. Agreement with earlier supernova data has been established in the presence of a cosmological constant. In this analysis the dark radiation from the bulk was switched off ($\Omega_d = 0$) and the energy-momentum squared term was kept. Under these assumptions, for flat spatial sections and matter parameter $\Omega_\rho = 0.3$ the maximum likelihood method gave $\Omega_\lambda = 0.004 \pm 0.016$ for the parameter characterizing the source term quadratic in the energy-momentum. This in turn implies a tiny value of the brane tension, which is disfavored by generic brane-world arguments. Moreover, much lower values for Ω_λ emerge from both CMB and BBN.

In contrast to Refs. [18] and [26] the analysis of [27] keeps both Ω_λ and Ω_d, the latter obeying $|\Omega_d| < 0.01$. The best fit is obtained at $\Omega_\rho = 0.15$, $\Omega_\Lambda = 0.80$, $\Omega_\lambda = 0.026$ and $\Omega_d = 0.008$. With the high value of the brane tension set by either (a) the value of the 4-dimensional Planck constant and sub-millimeter tests [28] on possible deviations from Newton's law (in units $c = 1 = \hbar$ these give $\lambda_{tabletop}^{\min} = 138.59\ \mathrm{TeV}^4$ see [29,7]), (b) astrophysical considerations $\lambda_{astro}^{\min} = 5\times 10^8\ \mathrm{MeV}^4$[30] or (c) BBN constraints $\lambda_{BBN}^{\min} = 1\ \mathrm{MeV}^4$ [31], the quadratic source term barely counts at late-times in the cosmological evolution.

Given the high limits for the values of λ, in any realistic model Ω_λ can be safely ignored. This is a crucial difference of our forthcoming analysis as compared to the one presented in Refs. [27] and [26], where the corresponding cosmological parameter Ω_λ was kept.

The next question is whether the source term arising from the Weyl curvature of the bulk may be kept, in other worlds, whether $\Omega_d \neq 0$. The Weyl curvature of the bulk gives an energy density

$\rho_d = 6m/\kappa^2 a^4$, where $\kappa^2 = 8\pi G$ is the gravitational coupling constant. In Ref. [32] it was shown that the BBN limits constrained the dark radiation component as $-1.23 \le \rho_d(z_{BBN})/\rho_\gamma(z_{BBN}) = 0.11$. Combining this with CMB constraints reduces this range to $-0.41 \le \rho_d(z_{BBN})/\rho_\gamma(z_{BBN}) \le 0.105$. Here ρ_γ is the energy density of the background photons. Another constraint for the value of the dark radiation at BBN was derived in [33] as $-1 < \rho_d(z_{BBN})/\rho_\nu(z_{BBN}) < 0.5$ where ρ_ν is the energy density contributed by a single, two-component massless neutrino. This constraint was derived for high values of the 5-dimensional Plank mass.

In the simplest case the Weyl source term evolves as a radiation, thus its present value is obviously tiny. This is the reason why all mentioned references [18] and [26] comparing RS brane-worlds with observations disregard dark radiation. But is this a necessary assumption? Formulating the question the other way around: if we include even a small component of dark radiation into the late-time universe model we face a serious problem. Due to the fact that the energy density of dark radiation decreases as a^{-4} (compared to that of matter which is a^{-3}), even an amount of dark radiation of the same order as the amount of baryonic matter nowadays implies dark radiation dominance in the past, for example during structure formation. This conclusion is contradicted by numerical simulations, which favorize cold dark matter as the dominant component of the Universe during structure formation [34].

However we can generalize the validity of the model by lifting the requirement of a *constant* mass m in the dark radiation energy density. A constant m implies a static Schwarzschild-anti de Sitter bulk and no energy exchange between the brane and the bulk. Therefore dark radiation is a manifestation of an equilibrium configuration with a static bulk, and it may be well possible that such a situation is reached only at the latest stages of the evolution of the brane-world Universe. Whenever m depends on a certain, non-zero power of a, the evolution of the energy density of the Weyl source term evolves in a non-standard way, allowing to escape from the argument of a small Weyl fluid left nowadays.

We propose here the LWRS (Lambda-Weyl fluid-Randall-Sundrum) model, a specific *RS model with i) cosmological constant, ii) the brane radiating away energy during various stages of the cosmological evolution, characterized by the index a and iii) a Weyl fluid depending on the actiul value of a during the latest stage of cosmological evolution*, which can be tested by supernova observations. For the inclusion of the LWRS model in the classification of brane-world models, see Fig 1 of paper I.

The LWRS model takes into account the possibility of an energy exchange between the brane and the bulk. This idea is not new. Indeed, it was already proposed that during an inflationary phase on the brane radiation is emitted and black holes thermally nucleate in the bulk [35]. Later on, but still in the high energy regime, the brane radiates such that the *mass function* of the bulk black hole increases with a^4 [36]. This means that the Weyl source term becomes a constant in this era. The brane continues to radiate away energy during structure formation [37], a process leading to a bulk black hole mass function $m \propto a^{\alpha}$, with $1 \leq \alpha \leq 4$. (Other models with the brane radiating energy into the bulk are also known [38].)

For $\alpha = 0$ the Weyl fluid is known as dark radiation, for $\alpha = 2, 3$ it gives the correct growth factor during structure formation. For $\alpha = 1, 4$ it is indistinguishable from dark matter and a cosmological constant, respectively. Therefore pure dark radiation can emerge only in the low-z limit, while at earlier times a dynamic bulk – brane interaction governed by energy exchange should be present.

2. Confronting the models with the selected supernova data

As type Ia supernovae result from the explosion of white dwarf stars with identical mass, they show remarkable similarities. By employing well established calibration methods, one can calculate the maximal luminosity of the object (in the reference system of the explosion). This is done by analyzing the time-dependent variation of the emitted luminosity and the spectrum, a method known as the Multi-Color Light Curve analysis [39,23]. In this process the observed parameters, the *shape of the light curve* and the *spectral distribution of the emission* have to be converted into the reference system of the host galaxy. For distant supernovae this translates to take into account the time dilation and the so-called K-correction [40]. While these methods depend on z, they are independent on the specific cosmological model. After performing these corrections, we have well-calibrated maximal luminosities for the supernovae of type Ia and in consequence they are considered as standard candles.

In 2003 a list of d_L-z data pairs were published for 230 supernovae of type Ia [41], and 60 of them had low absoprtion (i.e. A_V.1) and cosmological redshift ($z > 0.01$). To give result which can easily be compared to earlier works, we also involve this selected low-absorption data set for the most of the examinations. The basic supernova data we use here is the improved Gold set [42], which was released in 2006.

We confront with supernova observations several models from paper I. In Fig 1 we represent graphically on both logarithmic and linear scales their luminosity distance -redshift relations up to $z = 2.5$. The plots are for $k = 0$ and $\Omega_\rho = 0.27$ (according to the combined analysis of the SDSS and WMAP 1-year data in Ref [2]). In particular, the luminosity distance – redshift relation is shown for the following models:

• The LWRS model (the perturbative solution given by Eqs. (56), (60)–(61), (63), (65) and (67) of paper I, with $\Omega_\lambda = 0$ and for $\alpha = 0$) for the two values of the late-time dark radiation $\Omega_d =$ -0.05 and $\Omega_d = 0.05$ (the curves 1 and 3, respectively). The latter models contain a brane which radiates energy at early times (for $\Omega_d > 0$) and during structure formation, such that a bulk black hole is formed and its mass increases continuously. As this process slows down, the Weyl curvature of the bulk induces the late-time dark radiation on the brane.

• The ΛCDM model, given by Eqs. (60)–(61) of paper I (curve 2).

• The solutions with brane tension $\lambda = 2\Lambda/\kappa^2$ and no dark radiation (given by Eq. (52) of paper I) for both admissible values for this model, at $\Omega_\Lambda = 0.704$ (curve 4) and $\Omega_\Lambda = 0.026$ (curve 6). The former is similar to the class of models discussed in [26].

• The late-time universe $\Omega_\Lambda = 0$ limit of the RS model with Randall-Sundrum fine-tuning, containing a huge amount of dark radiation $\Omega_d = 0.73$, given by Eq. (44) of paper I (curve 5).

In Fig. 1 we plot these models in a comparison to low-absorption supernova data from Ref. [41] (red triangles) together with the Gold set [23] (black dots). The error bars are indicated in the respective colors. The diagrams with linear scale are more instructive, as they emphasize the difference among the predictions of the chosen models and how they fit data, while the logarithmic scale better disseminate between the low z points.

The models represented by the curves 1, 3 and 4 by eye seem to compare as well with the supernova observations as the ΛCDM model (curve 2). By contrast, the models represented by the curves 5 and 6 seem to be not supported by observations. The model with no cosmological constant and significant dark radiation $\Omega_d = 0.73$, $\Omega_\lambda = 0$ (curve 6) and the model with $\Lambda = \kappa^2\lambda/2$ and $\Omega_\Lambda = 0.025$ (curve 5) are significantly inconsistent with the observations, as they give $\chi^2 = 213$ and 395, respectively[‡]. All other models shown on Fig 1 are comparable with the supernova observations, as it was expected by a simple glance.

The $\chi^2 = 50$ value found for the $\Lambda = \kappa^2\lambda/2$-model with $\Omega_\Lambda = 0.74$ (curve 4) is slightly better than χ^2 found the ΛCDM model. However, as mentioned earlier, the tiny brane tension $\lambda = 38.375 \times 10^{-60}\,\mathrm{TeV}^4$, several order of magnitudes lower than all existing lower limits rules out this model as well.

The best fitting models are the models with brane cosmological constant; a high value of the brane tension (leading to $\Omega_\Lambda \approx 0$) and a small contribution of dark radiation, $\Omega_d = \pm 0.05$ (the curves 1 and 3). For $\Omega_d = -0.05$ we find $\chi^2 = 65$, which is still acceptable. For $\Omega_d = 0.05$ we get $\chi^2 = 49$.

Values of Ω_d between these limits are also admissible. It is likely that by increasing Ω_d towards higher positive values, χ^2 remains compatible, however the accuracy of the perturbative solution is deteriorated with increasing Ω_d, therefore higher orders in the expansion would be necessary to take into account.

3. The Gold2006 set of supernovae

More recently, Riess et al. [42] have published a new set of 182 gold supernovae, including new HST observations and recalibrations of the previous measurements. It is an interesting question how this recalibration influenced the above conclusions for the well-fitting models with dark radiation.

We applied the same tests to the Gold2006 data set as described in the previous section. First we assumed that $\Omega_\rho = 0.27$, as before, cf. Ref [2]. In this case the critical value of χ^2 is 197 at 80% level and 209 at 90% confidence level. Then the models represented by the curves 1–4 of Fig 2 behave as follows. The model with a small amount of negative dark radiation is disfavored at 80% confidence ($\chi^2 = 204$). The models with $\lambda = 2\Lambda/k^2$ and $\Omega_\Lambda = 0.704$ are ruled out at 90% confidence level, too (as $\chi^2 = 221$). As expected from the previous analysis, the ΛCDM model ($\chi^2 = 192$) and the LWRS model with $\Omega_d = 0.05$ (giving $\chi^2 = 194$) compete closely. We also remark that varying Ω_d between -0.03 and 0.07, the χ^2 remains under the critical value.

For gaining a deeper insight we have then calculated the predictions of the models between $\Omega_d = -0.10 \div 0.10$ with a stepsize of 0.01 in Ω_d, with Ω_ρ allowed to freely vary in the domain $0.15 \div 0.35$ and z in the range $0 \div 3$. Then we looked for the best fit of the Gold2006 set in the Ω_d - Ω_ρ space. This is represented on Fig 3. The global minimum of the surface is at $\Omega_d = 0.040$, $\Omega_\rho = 0.225$ ($\chi^2 = 190.52$), which suggests an interesting opportunity for a Universe with less baryonic density and with dark radiation, compatible with the Gold2006 supernova data. The 1-σ confidence

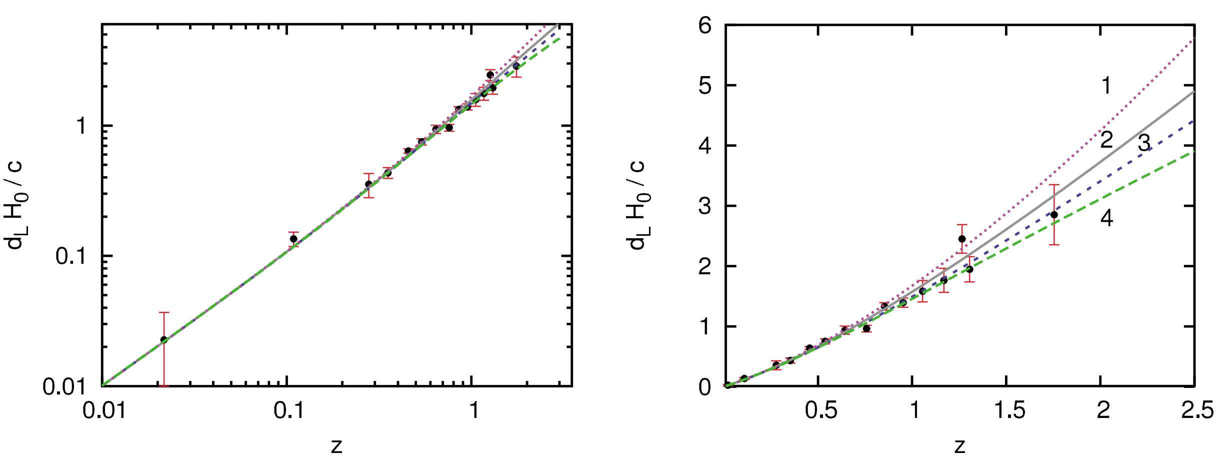

Figure 2
(Color online) The luminosity distance – redshift relation for the viable brane-world models and the ΛCDM model (the curves (1)–(4) of Fig 1), both with logarithmic (left panel) and linear scale (right panel), compared to the smeared Gold set [23]. The best fit is obtained for the brane-world model (3), with 5% dark radiation.

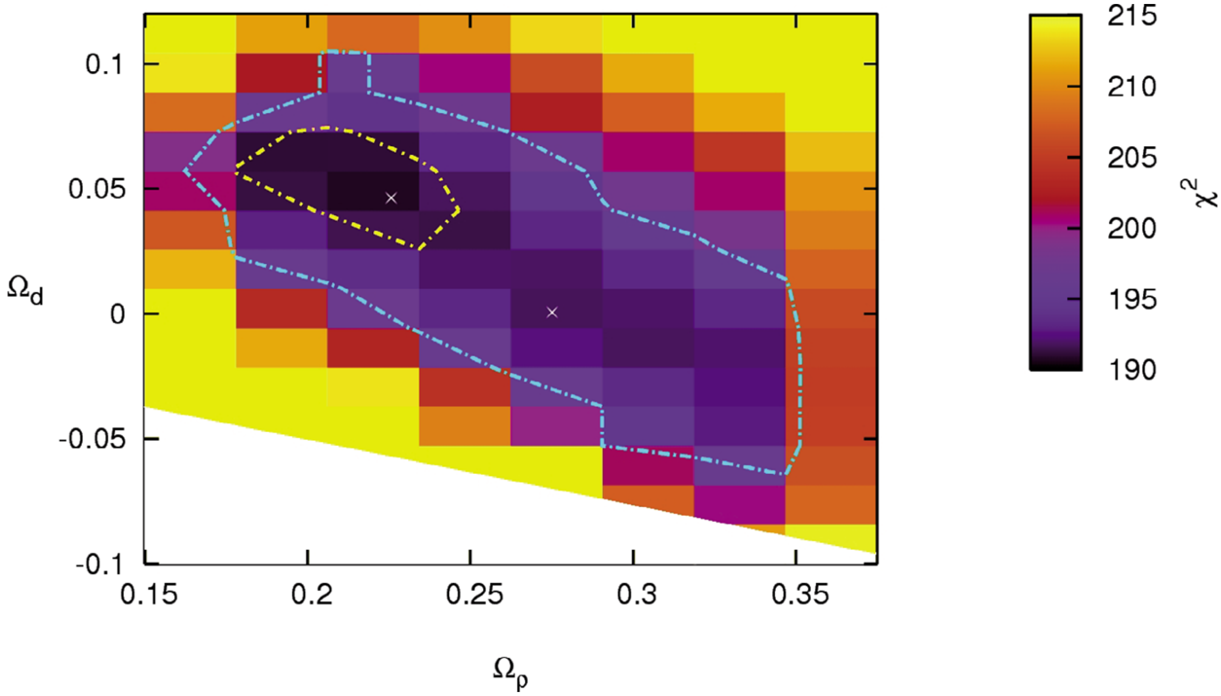

Figure 3
(Color online) The fit of the luminosity distance – redshift relation for the LWRS brane-world models with dark radiation, (including the ΛCDM model for $\Omega_d = 0$). There is no assumption for $\Omega_\rho \in (0.15, 0.35)$, its preferred value 0.225 being determined from the supernova data, together with the preferred value 0.040 of Ω_d. The contours refer to the 1-σ and 2-σ confidence levels and both are centered on the LWRS model with the values given above. The local minimum represented by the ΛCDM model is at $\Omega_\rho = 0.275$. Both the global and local minima are marked. The white area in the lower left corner represents the forbidden region of the parameter space for z = 3.

interval is centered about this value. The ΛCDM model (where Ω_d is exactly 0) has the local minimum of $\Omega_\rho = 0.275$ ($\chi^2 = 195.8$), but this is outside the 1-σ confidence interval.

Similar conclusions emerge from the plot in the Ω_Λ - Ω_ρ plane, Fig 4. Here the global minimum of the surface is at $\Omega_\Lambda = 0.735$, $\Omega_\rho = 0.225$. The local minimum of the CDM model is at $\Omega_\Lambda = 0.725$.

We note that there is a forbidden parameter range in both planes Ω_d - Ω_ρ and Ω_Λ - Ω_ρ, represented by white regions on Figs 3 and 4. This is because the Friedmann equation for these brane-world models

$$\left[\frac{H(z)}{H_0}\right]^2 = \Omega_\Lambda + \Omega_\rho(1+z)^3 + \Omega_d(1+z)^4 > 0, \tag{1}$$

combined with $\Omega_\Lambda + \Omega_\rho + \Omega_d = 1$ gives the constraints

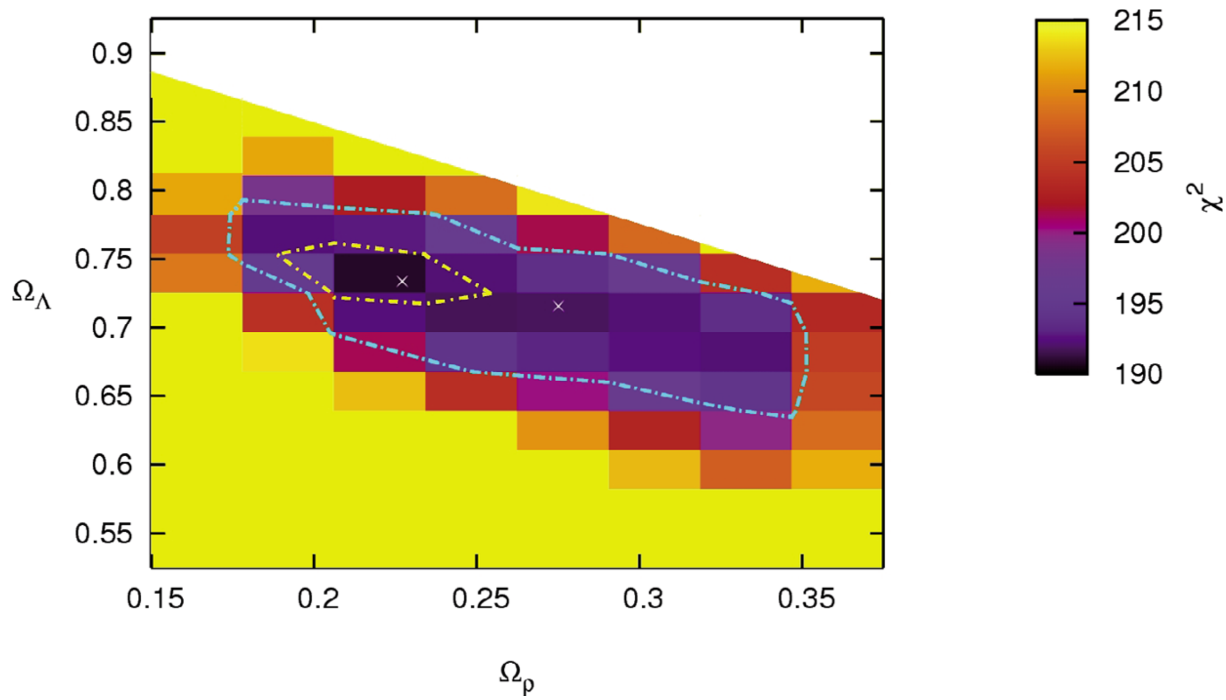

Figure 4
(Color online) Same as on Fig 3, but in the Ω_Λ - Ω_ρ plane. The global minimum is at $\Omega_\Lambda = 0.735$, $\Omega_\rho = 0.225$, while the local minimum for the ΛCDM model gives $\Omega_\Lambda = 0.725$ and $\Omega_m = 0.275$ (both marked). The white area on the top right corner represents the forbidden parameter range.

$$\Omega_d [(1 + z)^4 - 1] + \Omega_\rho [(1 + z)^3 - 1] + 1 > 0 \tag{2}$$

in the Ω_d - Ω_ρ plane and

$$\Omega_\Lambda [(1 + z)^4 - 1] - (1 + z)^3 [1 + z(1 - \Omega_\rho)] < 0 \tag{3}$$

in the Ω_Λ - Ω_ρ plane.

The forbidden region increases in both cases with z. If we would like to extend the limits to $z \to \propto$, we obtain the limiting curves $\lim_{z\to\infty} \Omega_d^{\min}(z, \Omega_\rho) = 0$ in the Ω_d - Ω_ρ plane and $\lim_{z\to\infty} \Omega_\Lambda^{\max}(z, \Omega_\rho) = 1 - \Omega_\rho$ in the Ω_Λ - Ω_ρ plane. However the LWRS model being valid only for low values of z, we represent on the graphs only the forbidden range for $z = 3$.

4. The compatibility of the LWRS model $\Omega_d = 0.04$ and $\alpha = 0$ with cosmological evolution

The energy density of dark radiation decreases too fast during cosmological evolution to result in a considerable amount nowadays. We discuss this problem and its possible remedy in detail here. First we comment on the problem, then we show how an energy exchange between the brane and the bulk can leave a considerable amount of dark radiation.

The constraint derived in [32] for the energy density of the dark radiation:

$$-0.41 \leq \frac{\rho_d(z_{BBN})}{\rho_\gamma(z_{BBN})} \leq 0.105, \tag{4}$$

where $\rho_\gamma(z_{BBN}) = \beta T_{BBN}^4$ is the energy density of the background photons at the beginning of BBN. The coefficient

$$\beta = \frac{\pi^2}{30} g_* \frac{k_B^4}{(\hbar c)^3} = 3.78 \times 10^{-16} g_* \text{ J m}^{-3} \text{ K}^{-4} \tag{5}$$

contains [11,43] the effective number g_* of relativistic degrees of freedom, which depends on the temperature. According to [44]g_* = 10.75 at the beginning of BBN, when T_{BBN} = 1.16 × 10^{10} K. Thus $\rho_\gamma(z_{BBN})$ = 7.37 × 10^{25} J m^{-3} emerges, giving the constraint

$$-3.02 \times 12^{25} \text{ Jm}^{-3} \leq \rho_d(z_{BBN}) \leq 7.74 \times 10^{24} \text{ Jm}^{-3}. \tag{6}$$

Note, that the domain of allowable negative values is larger than the one for positive values.

As for today the background photons have cooled to T_0 = 2.725 K and for such low temperatures g_* = 3.36 [43,44] their energy density $\rho_\gamma = \rho_\gamma(z = 0)$ is

$$\rho_\gamma = 7.01 \times 10^{-14} \text{ Jm}^{-3}. \tag{7}$$

With the value $H_0 = 73^{+3}_{-3}$ Km s^{-1} Mpc^{-1} of the Hubble constant [3], cf. Eq. (23) of paper I the present day cosmological parameters ρ and Ω (both for background and dark radiation) relate as

$$\rho_{d,\gamma} = 9.00 \times 10^{-10} \Omega_{d,\gamma} \text{Jm}^{-3}. \tag{8}$$

Thus the present value of Ω_γ is

$$\Omega_\gamma = 7.74 \times 10^{-5}, \tag{9}$$

which is quite negligible. If the Weyl source term were to evolve as radiation, its value would be even smaller, cf. Eq. (4). Indeed Eqs. (6) and (8) imply

$$-1.02 \times 10^{-4} \leq \Omega_d \leq 2.62 \times 10^{-5}. \tag{10}$$

$|\Omega_d|$ is of the same order of magnitude or smaller as Ω_γ.

However if the brane is radiating during structure formation, the mass parameter m becomes a function of the scale factor $m \propto a^\alpha$, with $1 \leq \alpha \leq 4$ [37]. Then the energy density scales as $a^{4-\alpha}$.

Now let us suppose that the brane is in an equilibrium (non-radiating) configuration with $\alpha = 0$ in the domain $0 \leq z \leq z_1$. In a preceding era $z_1 < z \leq z_*$ the brane radiates such that $\alpha \neq 0$, finally right after the beginning of BBN, at $z_* < z \leq z_{BBN}$ there is equilibrium once more ($\alpha = 0$). Here $z_{BBN} = (T_{BBN}/T_0) - 1 = 4.26 \times 10^9$. According to this evolution

$$\begin{aligned} \rho_d(z_{BBN}) &= \rho_d \left(\frac{a_0}{a_1} \right)^4 \left(\frac{a_1}{a_*} \right)^{4-\alpha} \left(\frac{a_*}{a_{BBN}} \right)^4 \\ &= \rho_d \left(\frac{1+z_1}{1+z_*} \right)^\alpha (1 + z_{BBN})^4. \end{aligned} \tag{11}$$

Inserting this in Eq. (6) and employing Eq. (8) we obtain:

$$-1.02\times10^{-4} \leq \left(\frac{1+z_1}{1+z_*} \right)^\alpha \quad \Omega_d \leq 2.62\times10^{-5}. \tag{12}$$

In the particular case $\alpha = 0$ we recover the constraint (10) set on pure dark radiation. However for any $\alpha > 0$ we get

$$z_* \geq (1 + z_1) \, [\max(-0.98\, \Omega_d, 3.82\, \Omega_d)]^{1/\alpha} \times 10^{4/\alpha} - 1. \tag{13}$$

Let us specify this result for the best fit value $\Omega_d = 0.04$. Depending on α we obtain the following numerical relations between the redshifts characterizing the switching on and off of the radiation leaving the brane:

$$z_* \geq \begin{cases} 1527.80 + 1528.80\, z_1 & , \quad \alpha = 1 \\ 38.10 + 39.10\, z_1 & , \quad \alpha = 2 \\ 10.52 + 11.52\, z_1 & , \quad \alpha = 3 \\ 5.25 + 6.25\, z_1 & , \quad \alpha = 4 \end{cases}. \tag{14}$$

It is evident that the value of z_* increases with z_1 (this dependence becoming an approximate scaling for higher values of z_1) and decreases with α.

The lower limit in the LWRS model is $z_1 = 3$. Then

$$z_* \geq \begin{cases} 6114.20 & , \quad \alpha = 1 \\ 155.40 & , \quad \alpha = 2 \\ 45.08 & , \quad \alpha = 3 \\ 24.01 & , \quad \alpha = 4 \end{cases}. \tag{15}$$

For the higher values of α the duration of the radiative brane regime necessary to produce a high value of Ω_d today is quite short.

5. LWRS models with α = 2, 3 confronted with supernova data

In the absence of a known mechanism for changing a, we examine here the cases when $\alpha = 2$ and $\alpha = 3$ hold throughout the cosmological evolution, up to nowadays. For this we confront these models with the Gold 2006 set exactly as described before. To preserve the validity of the perturbative solution, the range of Ω_d was selected to be -0.1–0.1, and we probed the range 0.15–0.35 of Ω_ρ. The assumption for flatness was kept, too.

The results are qualitatively similar to the $\alpha = 0$ case. The remarkable difference is that the peak of the minimum turned into a "trough", which lies aslope in the Ω_ρ - Ω_d space. This means that instead of a district solution, a complete model family exists in both cases, which can equally well explain the supernova data. The Ω_ρ dependence of Ω_d is less in the $\alpha = 2$ model as compared to $\alpha = 0$, and is very small if $\alpha = 3$. The $\Omega_d = 0$ case is the ΛCDM model where these model intersect. The steeper slope of the minimum trough thus allows a much lower range of Ω_ρ in the $\alpha = 2$, and especially in the $\alpha = 3$ models, with a value close to 0.3. On the other hand, the range of Ω_d gets more and more wide with increasing α, which results in the conclusion that the presence of Ω_d is mathematically plausible, and they have to be accounted for in RS cosmology.

Due to the higher slopes of the 1-σ and 2-σ contours in these α = 2, 3 models, Ω_ρ is much less affected by the Weyl fluid, while Ω_d can have various values in the detriment of Ω_Λ. Therefore the Weyl fluid can explain some of the dark energy.

6. Conclusion

The luminosity distance given in paper I as function of redshift in terms of elementary functions and elliptical integrals of first and second type for various brane-world models with Weyl fluid was confronted with the available supernova data sets, including the Gold2006 data [42]. The tested models were:

(A) The models with Randall-Sundrum fine-tuning, discussed in section 4 of paper I, with a considerable amount of dark radiation as a bulk effect, and a high value of the brane tension.

(B) The two models discussed in subsection 5.1 of paper I, which obey $\Lambda = \kappa^2\lambda/2$, have no dark radiation and were integrable in terms of elementary functions.

(C) The LWRS models (subsection 5.2 of paper I), with a brane cosmological constant, for which the luminosity distance could be given analytically as function of redshift to first order accuracy in the dark radiation. (Due to its smallness, the source term Ω_λ quadratic in the energy density was suppressed in the perturbative models of paper I.)

The brane-world models (A) although interesting for historical reasons, do not comply with observations. Even if we introduce an extremely high amount of dark radiation $\Omega_d = 0.73$, tentatively replacing the cosmological constant in the energy balance $\Omega_\Lambda + \Omega_\rho + \Omega_d + \Omega_\lambda = 1$, these models are quickly outruled by supernova data (curve 5 of Fig 1). Dark radiation is not capable to replace the cosmological constant in producing a late-time acceleration, since it scales as usual radiation. The more we go back in the past, the higher becomes its domination over matter. Therefore a cosmological constant or dark energy is still needed in the generalized Randall-Sundrum type II models.

Our analysis has also dismissed immediately the model (B) with $\Omega_\Lambda = 0.026$. Surprisingly, the other toy model (B) with $\Omega_\Lambda = 0.704$ was in good agreement with the Gold2006 data, but ruled out by its low value of the brane tension, similarly as the models discussed in Ref. [26]. A low brane tension is in disagreement with various upper limits set by cosmological and astrophysical tests.

The perturbative approach of subsection 5.2 of paper I can be considered valid for a Weyl fluid with $-0.1 < \Omega_d < 0.1$. In this range the LWRS brane-world models (C) were confronted with supernova data and for $\alpha = 0$ the dark radiation with significant negative energy density ruled out. The fact that a positive dark radiation (corresponding to a bulk black hole rather than to a bulk naked singularity) is favoured by the presently available best supernova data is in accordance with the early behavior of the RS model with late-time dark radiation, where the brane radiating away energy in early times leads to a black hole, which can further grow during structure formation.

The remaining LWRS brane-world models with $\alpha = 0$ and Ω_d between -0.03 and 0.07 (and Ω_ρ changed accordingly) turned out to be excellent candidates for describing our universe, as they show remarkable agreement with the Gold2006 supernova data sets. If Ω_ρ is allowed to vary in the range (0.15, 0.35), the preferred values are $\Omega_d = 0.040$, $\Omega_\rho = 0.225$, $\Omega_\Lambda = 0.735$.

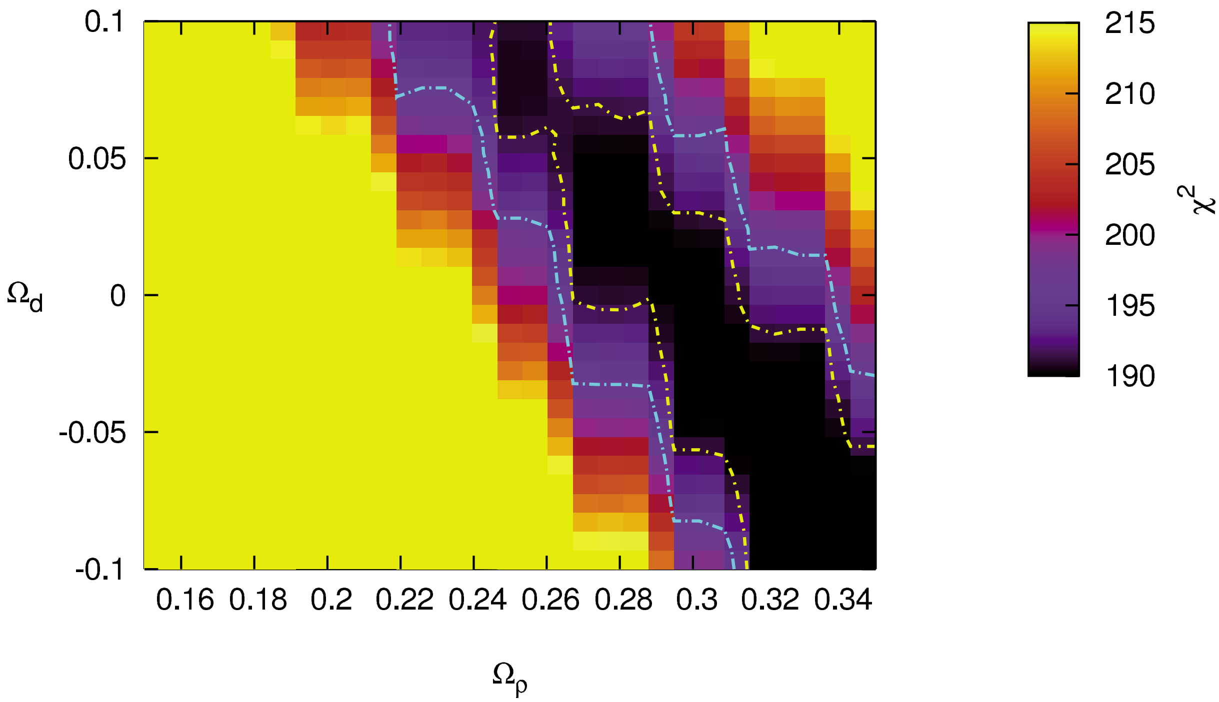

Figure 5
(Color online) Same as on Fig 3, but for the $\alpha = 2$ models.

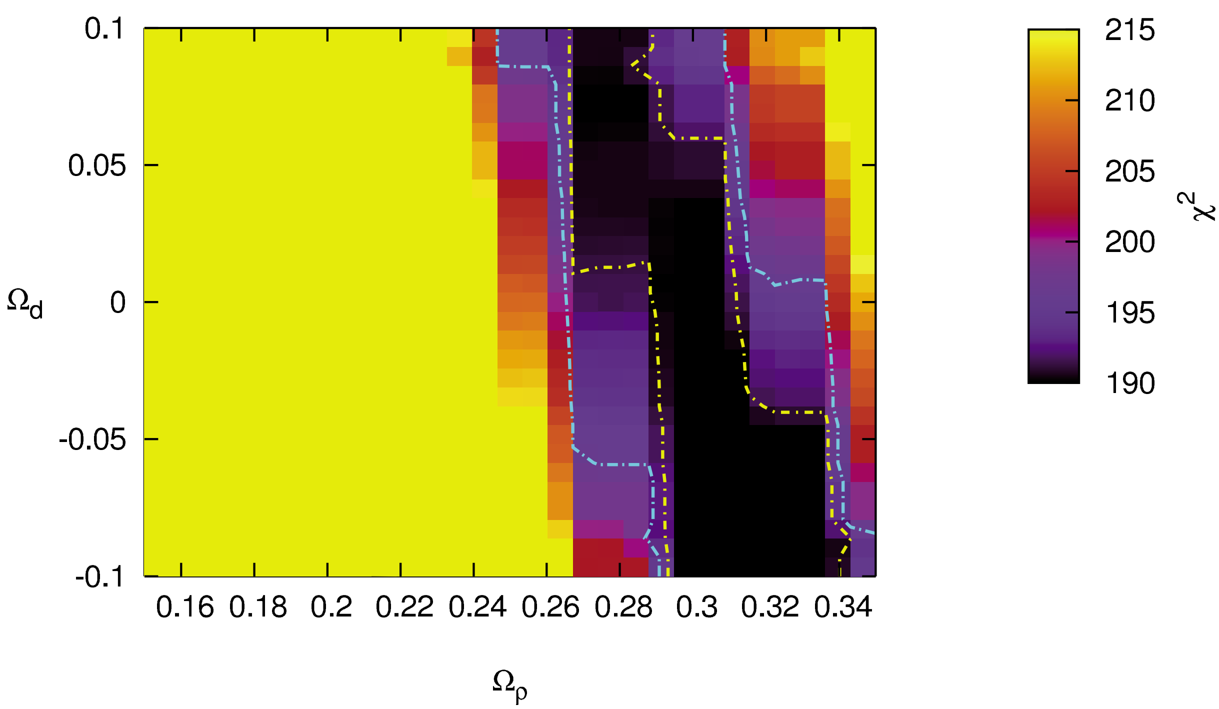

Figure 6
(Color online) Same as on Fig 3, but for the $\alpha = 3$ models.

The preferred cosmological parameters determined by comparing the LWRS model with α = 0 with supernova data alone are in perfect accordance with the WMAP 3-year data. Indeed according to Ref. [3] $\Omega_{\rho} h^2 = 0.127^{+0.007}_{-0.013}$ and $h = 0.73^{+0.03}_{-0.03}$ from which $\Omega_{\rho} = 0.238^{+0.035}_{-0.041}$ emerge. The value of Ω_{ρ} determined by comparing the LWRS model with the supernova data alone is well in the middle of the domain allowed by the WMAP 3 year data.

We have then proved that the preferred value of $\Omega_d = 0.04$ is compatible with the known history of the Universe if the brane radiates away energy into the bulk during a relatively short period of the cosmological evolution. Such a process occurring between $z = 24$ and $z = 3$ could increase the amount of dark energy today with a factor of 10^3 as compared to the non-radiating brane, exactly as required by the LWRS model with $\alpha = 0$.

The LWRS models with $\alpha = 1$ ($\alpha = 4$) are identical with the ΛCDM model with the only difference that some fraction of the dark matter (of the cosmological constant) has geometric origin.

Finally, the LWRS models with $\alpha = 2$ and $\alpha = 3$ do not present a sharp minimum, but rather an elongated trought shape in the parameter space, with the slope increasing with the value of α and $\Omega_{\rho} \approx 0.3$. This means that in this class of models a wide range of values for Ω_d (with a slight preference for negative values) and corresponding values for Ω_{Λ} are fitting to the supernova data.

We must note that the reliability of these values is somehow deteriorated by the relatively small number of high-z supernova and by the inherent difficulties in the calibration of the available data. An obvious source of error is that data from the Gold2006 set is a combination of measurements taken on different instruments [45] and in fact it has been already signaled that the Gold2006 data set is not statistically homogeneous [46].

The conclusion of this paper is somewhat similar to that of Ref. [22]: the presently available supernova data are not enough to discern among several cosmological models. However the difference between the predictions of the acceptable models of our analysis (the ΛCDM model, the LWRS brane-world with $\alpha = 0$ and $\Omega_d = 0.04$ and the models with Weyl fluid and $\alpha = 2, 3$) are increasing with z. One may reasonably hope that the very far ($z > 2$) supernovae, which will be discovered for sure in the following decade, will improve their comparison.

Note

‡ We mention here that we have also excluded several other models with Randall-Sundrum fine-tuning (not shown on Fig 1), which have either a very low value of the brane tension or a significant dark radiation. For example, the models with $\Omega_d = 0.0258835$, $\Omega_{\lambda} = 0.70412$ and $\Omega_d = 0.70412$, $\Omega_{\lambda} = 0.0258835$ gave $\chi^2 = 246$ and 415, respectively.

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