Research article

Dilaton and off-shell (non-critical string) effects in Boltzmann equation for species abundances

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Abstract

In this work we derive the modifications to the Boltzmann equation governing the cosmic evolution of relic abundances induced by dilaton dissipative-source and non-critical-string terms in dilaton-driven non-equilibrium string Cosmologies. We also discuss briefly the most important phenomenological consequences, including modifications of the constraints on the available parameter space of cosmologically appealing particle physics models, imposed by recent precision data of astrophysical measurements.

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1 Introduction

In a previous work [1] we have discussed a case study of dissipative Liouville-string cosmology, involving non critical string cosmological backgrounds, with the identification [2,3] of target time with the world-sheet zero mode of the Liouville field [4-7]. Such cosmologies were found to asymptote (in cosmic time) with the conformal backgrounds of [8-10], which are thus viewed as equilibrium (relaxation) configurations of the non-equilibrium cosmologies.

In terms of microscopic considerations, a model for such departure from equilibrium could be considered the collision of two brane worlds, which results [11,12] in departure from conformal invariance of the effective string theory on both the bulk and the brane world, and thus in need for Liouville dressing to restore this symmetry [4-7]. Dynamical arguments [13,14], then, stemming from minimization of effective potentials in the low-energy string-inspired effective field theory on the brane, imply the (eventual) identification of the zero mode of the Liouville

mode with (a function) of target time [2,3]. There is an inherent *time irreversibility* in the process, which is associated with basic properties of the Liouville mode, viewed as a local (dynamical) renormalization group (RG) scale on the world-sheet of the string. This implies *relaxation* of the associated dark energy of such cosmologies, and a gravitational friction, associated with the conformal theory central charge deficit. For a recent review we refer the reader to [11,12], where concepts and methods are outlined in some detail.

For our purposes in this letter we would like to concentrate on one interesting aspect of the non-critical string cosmologies, associated with the off-equilibrium effects on the Boltzmann equation describing relic abundances and the associated particle-physics phenomenology. Indeed, in conventional cosmologies, a study of relic abundances by means of Boltzmann equation that governs their cosmic time evolution yields important phenomenological constraints on the parameters of particle physics (supersymmetric) models using recent (WMAP [15] and other) astrophysical Cosmic Microwave Background (CMB) data. Essentially, the astrophysical data constraint severely in some cases the available phase-space distributions of favorite supersymmetric dark matter candidates such as neutralinos [16-19]. When off-equilibrium, non-critical string cosmologies are considered [1], which notably are consistent with the current astrophysical data from supernovae, as demonstrated recently [20], then there are significant modifications to the Boltzmann equation, stemming from extra sources (dilaton) and off-shell (non-critical, non equilibrium) terms, which affect the time evolution of the phase-space density of the species under consideration.

It is the purpose of this paper to derive such modifications in detail, and then use them in order to discuss briefly particle-physics models constraints, especially from the point of view of supersymmetry. With regards to this last issue, in this article we shall present an approximate analytical treatment, which will only provide hints to what may actually happen. A complete analysis requires numerical studies which are postponed for a future publication.

2 Modified Boltzmann Equation in Non-Critical Strings

Consider the phase space density of a species X, which is assumed coupled to the off-shell (non-critical string) background terms:

$$f\left(\left|\vec{p}\right|, t\right)$$

where quantities refer to the Einstein frame, and the Einstein metric is assumed to be of Robertson-Walker (RW) type. Throughout this work we follow the normalization and conventions of [8-10].

For completeness we state the main relationships between the Einstein and σ-model frames [8]:

$$g_{\mu\nu} = e^{-2\Phi} g^{\sigma}_{\mu\nu}, \quad \frac{\partial t}{\partial t^{\sigma}} = e^{-\Phi} \tag{2.1}$$

where Φ is the dilaton field, and the superscript σ denotes quantities evaluated in the σ-model frame.

Since in the RW Einstein frame g_{00} = -1, and in the cosmic Einstein co-moving frame we have for a generic massive species with mass m (that we shall be interested in this work) p^{μ}_{σ} = $mdx^{\mu}/d\tau = m(dx^{\mu}/dt)\,(dt/dt_{\sigma}) = mp^{\mu}e^{-\Phi}$, it can be readily seen that

$$\left| \vec{p}^{\sigma} \right| \equiv \left(p^{i,\sigma}, p^{j,\sigma} g^{\sigma}_{ij} \right)^{1/2} = \left(p^i p^j g_{ij} \right)^{1/2} \equiv \left| \vec{p} \right| = a(t) \left(\sum_{i=1}^{3} p^i p^i \right)^{1/2} \tag{2.2}$$

We shall be interested in the action of the relativistic Liouville operator $\hat{L}$, acting on a phase-space density of species X, $f(|\vec{p}|, t, g^i)$, which in general depends on the off-shell backgrounds g^i = (g_{ii}, Φ), where g_{ii} are the spatial components of the RW space-time metric in Einstein frame, and Φ is the dilaton. The off-shell dependence comes about due to the interpretation of target time as a local world-sheet renormalization-group scale, the Liouville field.

We commence our analysis by recalling the relativistic form of the Liouville operator in conventional general relativity:

$$\hat{L}[f] = \left(p^{\alpha} \frac{\partial}{\partial x^{\alpha}} + \Gamma^{\alpha}_{\beta\sigma} p^{\beta} p^{\sigma} \frac{\partial}{\partial p^{\alpha}} \right) f \tag{2.3}$$

The second term on the r.h.s. is the relativistic form of the force, following from the geodesic equation. For a RW Universe, only the time-energy part survives from the first term, i.e.

$$p^{\alpha} \frac{\partial}{\partial x^{\alpha}} = E \frac{\partial}{\partial t}. \tag{2.4}$$

Moreover, the connection part receives non trivial contributions only from the terms $\sum_i \Gamma^0_{ii} p^i p^i \frac{\partial}{\partial E}$, i = 1, 2, 3 a spatial index (assuming for concreteness an already compactified string theory, or a theory on a three-brane world), with $\Gamma^0_{ii} = -a\dot{a}$, where the overdot denotes derivative w.r.t. cosmic RW time t, identified in our string theory with the Einstein-frame RW time (2.1).

Hence, the conventional part of the Liouville operator in a RW Universe would read [21]:

$$\hat{L}_{\text{conv}} = E\frac{\partial}{\partial t} - a\dot{a}\sum_{i=1}^{3} p^i p^i \frac{\partial}{\partial E} = E\frac{\partial}{\partial t} - \frac{\dot{a}}{a}\sum_{i=1}^{3}|\vec{p}|^2 \frac{\partial}{\partial E} \tag{2.5}$$

The presence of off-shell, non-critical (Liouville) string [4-7] backgrounds, upon the (dynamical) identification [2,3,11-14] of the Liouville mode with the target time, one will receive extra contributions in the expression for the associated Liouville operator, stemming from the fact that the latter is nothing but a total time derivative.

In the context of our string discussion it is important to specify that the initial frame, from which we commence our discussion, is the so-called Einstein frame. It is in this frame that the usual RW cosmology is obtained in string theory [9], for which the expression (2.5) is valid.

To discuss the non-critical-string (off-shell) corrections to Boltzmann equation, it will be necessary to consider time derivatives in the σ-model frame. This is due to the fact that it is in this frame that the target time $X^0 \equiv t_\sigma$ is related simply to the Liouville mode φ in non-critical string theories [2,3],

$$\varphi + t_\sigma = 0. \tag{2.6}$$

This relation is obtained dynamically from minimization arguments of the low-energy effective potential of some physically interesting cosmological models, for instance those involving colliding brane worlds [11,13]. To be precise, the initial relation (2.6) derived in the specific model of [13] reads: $\varphi/\sqrt{2} + t_\sigma = 0$, with the factor of $\sqrt{2}$ arising from (logarithmic) conformal-field-theory considerations, and being crucial for yielding a Minkowski space-time in the scenario of [13], where the coordinate X^0 assumed an initial Euclidean signature, appropriate for a world-sheet path-integral quantization. Nevertheless, in our approach below we shall absorb the $\sqrt{2}$ into the normalization of the units of the Regge slope α', for convenience.

In such scenarios the target-space dimensionality of the string is extended to $D + 1$ initially, with two time-like coordinates, t and φ. Eventually, these two coordinates are identified (c.f. (2.6)) in the solution of the generalized conformal invariance conditions that express the restoration of the conformal invariance by the Liouville mode, as we discuss below. This implies that, initially, the Liouville operator (2.3) acquire extra Liouville components (φ, p^φ), with $p^\varphi \equiv E_\varphi$. Hence, there are additional structures on the right hand side of (2.3), of the form $E_\varphi \frac{\partial}{\partial \varphi}$, together with the corresponding amendments in the Christoffel-symbol dependent $\partial/\partial p^\varphi$ terms, stem-

ming from the extra Liouville coordinate on the target space time, which implies appropriate extra components of the extended-target-space-time metric. The (dynamical) implementation of (2.6) restricts oneself to a *D*-dimensional hypersurface in the extended target-space, with the identification of $E_\varphi \to E$, the ordinary (physical) energy. From now on we restrict ourselves on this hypersurface, bearing in mind however that the identification of the Liouville mode φ with the target time should only be implemented at the end, and thus any dependence on φ should be kept explicit at intermediate steps of the pertinent calculations.

The connection between time derivatives in the Einstein and stringy-σ-model frames is provided by the chain rule of differentiation $\frac{\partial}{\partial t_\sigma} = \frac{\partial t}{\partial t_\sigma}\frac{\partial}{\partial t}$, using (2.1). This will result in multiplicative factors of $e^{-\Phi}$ in front of the appropriate non-critical-string modifications of the Liouville operator (2.5).

Since, as mentioned above, the Liouville operator is essentially a total time derivative operator, and in our case time is related to a world-sheet renormalization group scale, $\rho(\sigma, \tau)$, taken to be local on the two-dimensional surface as a result of world-sheet general covariance [22,23], the sought-for non-critical-string modifications of the Boltzmann equation emerge from the implicit dependence of the g^i background fields on $\rho(\sigma, \tau)$, that is the corresponding β-functions, which in a critical-string theory would vanish.

We now remark that in the approach of [2,3], the local RG scale $\rho(\xi)$ was identified with the dynamical Liouville mode φ, which implied that in this approach the local conformal invariance of the world-sheet of the string was restored [4-7],

$$\rho \equiv \varphi. \tag{2.7}$$

With these in mind we then modify the relativistic form (2.3), (2.4) by replacing

$$\frac{\partial}{\partial t} \to \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \int_\Sigma \frac{\partial t_\sigma}{\partial t}\frac{\partial \rho(\xi)}{\partial t_\sigma}\frac{\partial g^{"i"}}{\partial \rho(\xi)}\frac{\partial}{\partial g^{"i"}} =$$
$$\frac{\partial}{\partial t} + \eta e^\Phi \; \beta^{"i"} \; \frac{\partial}{\partial g^{"i"}} , \tag{2.8}$$

where $\xi = \sigma, \tau$, denotes the world-sheet coordinates, $\int_\Sigma$ is a world-sheet integration, and the index "*i*" runs on both, a (discrete) background field space, $\{g_{ii}(\gamma), \Phi(\gamma)\}$, $i = 1, 2, 3$ and a continuous *D*-dimensional (target) space-time γ. Hence, summation over "*i*" includes integration $\int d^D\gamma\sqrt{-g}\,(\dots)$. Note that since the (physical) target time in the σ-model frame, t_σ, is only the

world-sheet zero mode of the corresponding σ-model field $X^0(\xi)$, eventually the world-sheet integration $\int_\Sigma$ in (2.8) disappears, since only the world-sheet zero mode ρ of the local RG scale $\rho(\xi)$ (Liouville mode) will yield a non zero contribution. In the last equality of the right-hand side of (2.8), the quantity η is defined as: $\eta \equiv \frac{\partial \rho}{\partial t_\sigma}$. If we use the identification $\rho(\xi) = \varphi(\xi)$ and Eq. (2.6), then $\eta = -1$, but at this stage we keep it general to demonstrate explicitly the renormalization-scheme dependence (i.e. choice of the local scale $\rho(\xi)$).

The non-critical string contributions $\frac{\partial g^i}{\partial \rho} = \tilde{\beta}^i$ are the Weyl anomaly coefficients of the string, but with the renormalized σ-model couplings g^i being replaced by the Liouville-dressed [4-7] quantities. This is a particular feature of the approach of [2,3] viewing the Liouville mode as a local world-sheet RG scale, as mentioned previously. In turn the latter is also identified (c.f. (2.6)) with (an appropriate function of) the target time t.

The dynamics of this latter identification is encoded in the solution of the generalized conformal invariance conditions, after Liouville dressing, which read in the σ-model frame [2-7]:

$$-\tilde{\beta}^i = g^{i''} + Q g^{i'} \tag{2.9}$$

where the prime denotes differentiation with respect to the Liouville zero mode ρ, and the overall minus sign on the left-hand side of the above equation pertains to supercritical strings [8,9], with a time like signature of the Liouville mode, for which the central charge deficit $Q^2 > 0$ by convention.

Notice the dissipation, proportional to the (square root) of the central charge deficit Q, on the right-hand side of (2.9), which heralds the adjective *Dissipative* to the associate non-critical-string-inspired Cosmological model. Moreover, the Weyl anomaly coefficients $\tilde{\beta}^i$, $i = \{\Phi, g_{\mu\nu}\}$, whose vanishing would guarantee local conformal invariance of the string-cosmology background, are associated with off-shell variations of a low-energy effective string-inspired action, $\mathcal{S}[g^i]$

$$\frac{\delta \mathcal{S}[g]}{\delta g^i} = \mathcal{G}_{ij}\tilde{\beta}^j \quad \mathcal{G}_{ij} = \mathrm{Lim}_{z \to 0} z^2 \bar{z}^2 \langle V_i(z,\bar{z}) V_j(0,0) \rangle \tag{2.10}$$

with z, $\bar{z}$ (complex) world-sheet coordinates, $\mathcal{G}_{ij}$ the Zamolodchikov metric in theory space of strings, and V_i the σ-model vertex operators associated with the σ-model background field g^i. It is this off-shell relation that characterizes the entire non-critical (Q) Cosmology framework, asso-

ciated physically with a non-equilibrium situation as a result of an initial cosmically catastrophic event, at the beginning of the (irreversible) Liouville/cosmic-time flow [11,12].

The detailed dynamics of (2.9) are encoded in the solution for the scale factor $a(t)$ and the dilaton Φ in the simplified model considered in [1], after the identification of the Liouville mode with the target time (2.6). In fact, upon the inclusion of matter backgrounds, including dark matter species, the associated equations, after compactification to four target-space dimensions, read in the Einstein frame [1]:

$$3\,H^2 - \tilde{\varrho}_m - \varrho_\Phi = \frac{e^{2\Phi}}{2}\tilde{\mathcal{G}}_\Phi$$

$$2\,\dot{H} + \tilde{\varrho}_m + \varrho_\Phi + \tilde{p}_m + p_\Phi = \frac{\tilde{\mathcal{G}}_{ii}}{a^2} \tag{2.11}$$

$$\ddot{\Phi} + 3H\dot{\Phi} + \frac{1}{4}\frac{\partial\hat{V}_{all}}{\partial\Phi} + \frac{1}{2}(\tilde{\varrho}_m - 3\tilde{p}_m) = -\frac{3}{2}\frac{\tilde{\mathcal{G}}_{ii}}{a^2} - \frac{e^{2\Phi}}{2}\tilde{\mathcal{G}}_\Phi.$$

where $\tilde{\varrho}_m(\tilde{p}_m)$ denotes the matter energy density (pressure), including dark matter contributions, and $\varrho_\Phi\,(p_\Phi)$ the corresponding quantities for the dilaton dark-energy fluid. All derivatives in (2.11) are with respect the Einstein time t which is related to the Robertson-Walker cosmic time t_{RW} by $t = \omega t_{RW}$. Without loss of generality we have taken $\omega = \sqrt{3}\,H_0$ where H_0 is the Hubble constant. With this choice for ω the densities appearing in (2.11) are given in units of the critical density. Then one can see for instance that if the time t_{RW} is used the first of equations (2.11) above receives its familiar form in the RW geometry when the contributions of the dilaton and non-critical terms are absent.

The overdots in the above equations denote derivatives with respect to the Einstein time. Their right-hand side contain the non-critical string *off-shell* terms:

$$\tilde{\mathcal{G}}_\Phi = e^{-2\Phi}(\ddot{\Phi} - \dot{\Phi}^2 + Qe^\Phi\dot{\Phi})$$

$$\tilde{\mathcal{G}}_{ii} = 2\,a^2(\ddot{\Phi} + 3H\dot{\Phi} + \dot{\Phi}^2 + (1-q)H^2 + Qe^\Phi(\dot{\Phi} + H)). \tag{2.12}$$

In the above equations $H = (\dot{a})/a$ is the Hubble parameter and q is the deceleration parameter of the Universe $q \equiv -\ddot{a}a/\dot{a}^2$, and are both functions of (Einstein frame) cosmic time. The potential $\hat{V}_{all}$ appearing above is defined by $\hat{V}_{all} = 2Q^2\exp(2\Phi) + \exp(4\Phi)V$ where, in order to cover more general cases, we have also allowed for a potential term in the four-dimensional action $-\int d^4y\sqrt{-G}V$ in addition to that dependent on the central charge deficit term Q. Although we have assumed a (spatially) flat Universe, the terms on the r.h.s., which manifest departure from

the criticality, act in a sense like curvature terms as being non-zero at certain epochs. The dilaton energy density and pressure are given in this class of models by:

$$\varrho_\Phi = \frac{1}{2}(2\dot{\Phi}^2 + V_{all})$$

$$p_\Phi = \frac{1}{2}(2\dot{\Phi}^2 - V_{all}). \tag{2.13}$$

Notice that the dilaton field is not canonically normalized in this convention and its dimension has been set to zero.

For completeness, we mention at this point that the dependence of the central charge deficit $Q(t)$ on the cosmic time stems from the running of the latter with the world-sheet RG scale [2,3,11,12], and is provided by the Curci-Paffuti equation [24] expressing the renormalizability of the world-sheet theory. To leading order in an α' expansion, which we restrict ourselves in [1] and here, this equation in the Einstein frame reads:

$$\frac{d\tilde{\mathcal{G}}_\Phi}{dt_E} = -6\, e^{-2\Phi}(H + \dot{\Phi})\frac{\tilde{\mathcal{G}}_{ii}}{a^2}. \tag{2.14}$$

For future use we also state here the corresponding continuity of the matter stress tensor, which is not an independent equation, but can be obtained from (2.11) by appropriate algebraic manipulations:

$$\frac{d\tilde{\varrho}_m}{dt_E} + 3H(\tilde{\varrho}_m + \tilde{p}_m) + \frac{\dot{Q}}{2}\frac{\partial V_{all}}{\partial Q} - \dot{\Phi}(\tilde{\varrho}_m - 3\tilde{p}_m) = 6\,(H + \dot{\Phi})\frac{\tilde{\mathcal{G}}_{ii}}{a^2}. \tag{2.15}$$

This expresses the non-conservation equation of matter as a result of its coupling to *both*, the dilaton source terms and the off-shell, non-equilibrium (non-critical-string) backgrounds.

A consistent solution of $a(t)$, $\Phi(t)$, and the various densities, including back reaction of matter onto the space-time geometry, has been discussed in [1], where we refer the interested reader for further study. Also note that a preliminary comparison of such non-critical strings theories with astrophysical data, demonstrating consistency at present, is given in [20]. It should be remarked at this point that the solutions obtained in this study tend asymptotically to those presented in [8-10]. In such an approach, the dilaton rolls down asymptotically in (Einstein) target time as Φ = const. - ln t, consistent with a vanishing string coupling $g_s = e^\Phi \to 0$ as $t \to +\infty$. This situation is opposite to other studies in critical (equilibrium) string cosmologies, where the dilaton gets a non-trivial vacuum expectation value (v.e.v.) through an appropriate minimisation of its potential, being thus stabilised at early times. In such on-shell situations, as opposed to our non-equilibrium (off-shell) model studied in this and in previous articles [1,25,26], matter particles may

acquire dilaton-v.e.v.-dependent masses, which may be heavy, and as such they decouple from a thermal bath. In contrast, as we shall discuss in the next section, our non-equilibrium rolling dilaton and running string coupling, implies that at late eras of the Universe, one may still evaluate the relic abundances of matter particles as in conventional cosmology, using an appropriately modified Boltzmann equation.

A final, but important comment should be made at this juncture. It is understood that, in order not to disturb the delicate balance between particle interactions and the expansion of the Universe during the nucleosynthesis era, it is necessary to consider models in which the dilaton may be almost constant during that era, in such a way that the associated string coupling and particle decay and/or recombination rates are not affected much by the dilaton and non-critical string terms, as compared with the conventional cosmology situation. This important and by far not complete issue is highly model dependent, and it relies on specific properties of the underlying microscopic string theory, involving detailed scenaria of compactification, moduli field stabilization *etc*. In the literatute there are toy brane-inspired, non-critical-string cosmological models, where such approximately constant dilatons during early epochs of the Universe could be realised explicitly [13,14,25]. We also remark that similar assumptions are made in generic dilaton quintessence scenaria of critical strings [27].

After this necessary digression we now come back to discussing the derivation of the dilaton-source and noncritical-string induced modifications to the Boltzmann equation, governing the cosmic evolution of the various species densities. It is important for the reader to bear in mind already at this stage that the Boltzmann equation does not contain any independent information from the dynamical equations (2.11), but it should be rather viewed as an effective way of describing the cosmic evolution of the density of a given species, consistent with the continuity equation (2.15) for the total matter energy density. We shall come back to this important point later on.

At the moment, let us concentrate first on the dilaton-source and non-critical-string background contributions to the Liouville operator (2.8). The presence of (time-dependent) dilaton source terms implies an explicit dependence of the phase space density of a species f on Φ, while the non-conformal on the world-sheet) nature of the metric and dilaton background, induce a Liouville mode ρ dependence through the corresponding backgrounds:

$$f\left(\left|\vec{p}\right|, t, \Phi(t, \rho), g_{\mu\nu}(t, \rho)\right). \tag{2.16}$$

The non-critical string terms can be expressed, as we have seen above, in terms of the corresponding Weyl anomaly coefficients, which are non zero as a result of departure from conformal invariance of the pertinent string background. Despite their non-linear looking appearance when expressed in terms of the Weyl anomaly coefficients (which depend on the Ricci tensor and second covariant derivatives of the Dilaton field), such terms acquire a particularly simple linear

form once the identification of time with the Liouville mode (2.6) is implemented, which in effect implies that the explicit solution of (2.9), (2.11) must be taken into account when discussing the Boltzmann equation.

With the above in mind, the form of the non-critical-string (off-shell) and dilaton-source corrections to the Liouville operator reads (expressing quantities in the Einstein frame, taking into account the fact that in the σ-model frame $g^{\sigma}_{\mu\nu}$ and Φ are treated as independent field-theory variables, and using (2.1)):

$$\hat{L}_{\text{off-shell, dil}} \equiv E\dot{\Phi}\frac{\partial}{\partial\Phi} + E\eta e^{\Phi}\tilde{\beta}^{"i"}\frac{\partial}{\partial g^{"i"}} = E\dot{\Phi}\frac{\partial}{\partial\Phi} + E\eta e^{\Phi}\left(\sum_{i=1}^{3}\frac{\partial g^{\sigma}_{ii}}{\partial\rho}\frac{\partial g_{ii}}{\partial g^{\sigma}_{ii}}\frac{\partial}{\partial g_{ii}} + \frac{\partial\Phi}{\partial\rho}\frac{\partial}{\partial\Phi}\right) =$$

$$E\dot{\Phi}\frac{\partial}{\partial\Phi} + \eta E e^{\Phi}\{2a^2\left(\frac{a'}{a} + \Phi'\right)\sum_{i=1}^{3}\frac{\partial}{\partial g_{ii}} + \Phi'\frac{\partial}{\partial\Phi}\}$$

$$(2.17)$$

where the overdot denotes the derivative with respect to the (Einstein frame) cosmic time t, and the prime the derivative with respect to ρ, and we used the fact that:

$$\frac{\partial g^{\sigma}_{ii}}{\partial\rho} = 2\, a^2 e^{2\Phi}(\frac{a'}{a} + \Phi'), \quad \frac{\partial g_{ii}}{\partial g^{\sigma}_{ii}} = e^{-2\Phi} \qquad (2.18)$$

with $g_{00} = -1$, $g_{ii} = a^2$, and $\frac{\partial g^{\sigma}_{ii}}{\partial\rho} = \tilde{\beta}^{\text{Grav}}_{ii}$ the corresponding Weyl anomaly coefficient, in order to express $\tilde{\beta}^{\text{Grav}}_{ii}$ in terms of quantities in the Einstein frame. We have also been careful to separate the dilaton dependence on $\rho(t)$ from that on the ordinary time t (prior to the identification (2.6)). We remind the reader, at this point, that the identification (2.6) should only be applied at the end of the calculations, and should be viewed as a specific choice of "renormalization" scheme. The "renormalization" scheme (2.6), i.e. $\eta = -1$, which is obtained by a physical principle (dynamics) in certain models of interest to us here [13,14], may thus be viewed in this context as the "physical scheme", where observational cosmology can be analyzed. In such a scheme there is a simplification of the complicated non-linear expressions encoded in the Weyl anomaly coefficients.

Taking into account the implicit dependence of $f(|\vec{p}|,\ t,\ \Phi)$ on g_{ii} through $|\vec{p}| = \left(\sum_{i=1}^{3}p^i p^i g_{ii}\right)^{1/2}$, it is evident that $\partial f/\partial g_{ii} = (\partial|\vec{p}|/\partial g_{ii})\partial f/\partial|\vec{p}| = \frac{p_i p_i}{2|\vec{p}|}\frac{\partial f}{\partial|\vec{p}|}$, from which we

can easily see that the final form of the action of the Liouville operator, including dilaton/source terms and non-critical string corrections, on the phase-space density f (2.16), is such that:

$$\left(\hat{L}_{\text{conv}} + \hat{L}_{\text{off-shell,dil}} \right) f = C[f],$$

$$\frac{\partial f}{\partial t} = \frac{\dot{a}}{a} \frac{|\vec{p}|^2}{E} \frac{\partial f}{\partial E} - \eta e^{\Phi} \left(\frac{a'}{a} + \Phi' \right) |\vec{p}| \frac{\partial f}{\partial |\vec{p}|} - \left(\dot{\Phi} + \eta e^{\Phi} \Phi' \right) \frac{\partial f}{\partial \Phi} + \frac{1}{E} C[f].$$

(2.19)

Upon considering the action of the above operator on the density of a given species X, $n \equiv \int d^3 p f$ we then arrive, after some straightforward momentum integration by parts, at the *modified* Boltzmann equation for a four-dimensional effective field theory after string compactification (or restriction on three-brane worlds), in the presence of non-critical (off-shell) string backgrounds and dilaton source terms:

$$\frac{dn}{dt} = \frac{\dot{a}}{a} \int d^3 p \frac{|\vec{p}|^2}{E} \frac{\partial f}{\partial E} - \left(\dot{\Phi} + \eta e^{\Phi} \Phi' \right) \int d^3 p \frac{\partial f}{\partial \Phi} - \eta e^{\Phi} \left(\frac{a'}{a} + \Phi' \right) \int d^3 p |\vec{p}| \frac{\partial f}{\partial |\vec{p}|} + \int d^3 p \frac{C[f]}{E} \Rightarrow$$

$$\frac{dn}{dt} + 3 \frac{\dot{a}}{a} n = 3\eta e^{\Phi} \left(\frac{a'}{a} + \Phi' \right) n + \int \frac{d^3 p}{E} C[f] - \left(\dot{\Phi} + \eta e^{\Phi} \Phi' \right) \int d^3 p \frac{\partial f}{\partial \Phi}$$

(2.20)

The collision term $C[f]$ assumes the usual form in conventional particle cosmology [21]. The reader is invited to compare the final equation for the cosmic time evolution of the density n in the second line of (2.20) to the continuity equation for the total energy density of matter (2.15). As discussed previously, the Boltzmann equation (2.20) should be compatible in the sense of leading to no extra information) with the conservation equation (2.15), as well as the (modified) Einstein equations (2.11).

In the above scenario, where the non-critical string contributions have been obtained through the identification of time with the (world-sheet zero-mode of the) Liouville field ρ (2.6), there is some *universality* in the coupling of *all* matter species, including dark matter, to these off-shell background terms, which may be traced to the equivalence of all species coupled to gravity. This is to be contrasted with the more general phenomenological case studied in [1], where scenarios involving only the coupling of the non-critical string backgrounds with the exotic dark matter species have been considered as well.

The final issue to be discussed in this section pertains to the form of the dependence of f on the dilaton source terms, which would survive a dilaton-driven critical-string cosmology case, such as the one considered in [27]. We constrain this form by requiring that in the Einstein frame there are two types of dependence on Φ: (i) *explicit*, of the form $e^{-4\Phi}$, arising from the fact that in our approach, the phase space density is constructed as a quantity in the σ-model frame of the string, which is then expressed in terms of quantities in the Einstein frame. As such, it is by defi-

nition (as a density) *inversely* proportional to the proper σ-model frame volume $V^\sigma = \int d^4x \sqrt{-g^\sigma} \propto e^{4\Phi}$ on account of (2.1); (ii) *implicit*, corresponding to a dependence on Φ through the Einstein-frame metric g_{ii} (2.1). Hence the general structure of *f* is of the form:

$$f(\Phi, \vec{p}, \vec{x}, g^\sigma_{\mu\nu} = e^{2\Phi} g_{\mu\nu}; t) \propto e^{-4\Phi} \mathcal{F}(|\vec{p}|, \vec{x}, t) \tag{2.21}$$

This implies that:

$$\int d^3p \frac{\partial f}{\partial \Phi} = -4 \int d^3p f + \sum_{i=1}^{3} \int d^3p \frac{\partial g_{ii}}{\partial \Phi} \frac{\partial f}{\partial g_{ii}} = -4n - 2 \int d^3p \sum_{i=1}^{3} g_{ii} \frac{\partial |\vec{p}|}{\partial g_{ii}} \frac{\partial f}{\partial |\vec{p}|} =$$
$$-4n - \int d^3p |\vec{p}| \frac{\partial f}{\partial |\vec{p}|} = -4n + 3 \int d^3p f(|\vec{p}|, t) = -n, \tag{2.22}$$

where in the last step we have performed appropriate partial (momentum-space) integrations.

The final form of the Liouville operation (2.20), then, reads:

$$\frac{dn}{dt} + 3\left(\frac{\dot{a}}{a}\right)n - \dot{\Phi}n = 3\eta e^\Phi \frac{a'}{a}n + 4\eta e^\Phi \Phi' n + \int \frac{d^3p}{E} C[f] \tag{2.23}$$

We now notice that non-critical terms can be expressed in terms of the Weyl anomaly coeffcients for the (σ-model) graviton and dilaton backgrounds as:

$$3\eta e^\Phi \frac{a'}{a} = 4\eta e^{-\Phi} \left(\frac{1}{8} g^{\mu\nu} \tilde{\beta}^{\text{Grav}}_{\mu\nu} - \frac{3}{4} \tilde{\beta}^\Phi e^{2\Phi} \right)$$
$$4\eta e^\Phi \Phi' = 4\eta e^\Phi \tilde{\beta}^\Phi \tag{2.24}$$

where we used the Einstein frame metric to contract indices, with $\tilde{\beta}^{\text{Grav}}$ denoting the graviton Weyl anomaly coefficient. In (2.24) we have taken into account that in the class of models of [1,25], we are concentrating in this work, $\tilde{\beta}^{\text{Grav}}_{00} = 0$. Thus the Boltzmann equation finally becomes

$$\frac{dn}{dt} + 3\left(\frac{\dot{a}}{a}\right)n - \dot{\Phi}n = \frac{1}{2}\eta \left(e^{-\Phi} g^{\mu\nu} \tilde{\beta}^{\text{Grav}}_{\mu\nu} + 2e^\Phi \tilde{\beta}^\Phi \right)n + \int \frac{d^3p}{E} C[f]. \tag{2.25}$$

It is important to recall once more that, upon Liouville dressing, which restores the conformal invariance of the model, the graviton and dilaton Weyl anomaly coefficients satisfy (2.9), which upon the identification of the Liouville mode with (a function of the cosmic) time yield (2.11).

We next remark that in the case with non-critical string backgrounds, consistency requirements between the two equations determine the back reaction effects of matter onto space time. For instance, in the absence of matter, in the specific (but generic enough) non-critical string model of [25], we have that the present-era dilaton and scale factor of the Universe are such that $\dot{Q} \simeq 0$, $\dot{\Phi} + \frac{\dot{a}}{a} = 0$. In this particular case the off-shell terms at late eras are practically zero, and the system has reached equilibrium. The corresponding Boltzmann equation for the dilaton species densities is then satisfied in agreement with the corresponding continuity equation (2.15) in absence of matter.

However, in the presence of matter, we know from the analysis of [1] that $\dot{\Phi} + \frac{\dot{a}}{a} \neq 0$, and the right-hand-side of the Boltzmann equation (2.20), with n now representing matter species, yields terms proportional to n. Consistency with the continuity equation (2.15), then, implies that the off-shell (non-critical string background) terms in the latter should be proportional to the total number density of matter species (back reaction effects). This and other similar consistency checks should be made when considering solving such non-equilibrium cosmologies.

3 Phenomenology of Particle Physics Models and Modified Boltzmann Equation

In this section we consider solutions of the modified Boltzmann equation (2.23), or equivalently (2.25), for a particle species density n in the physically interesting case of supersymmetric dark matter species, such as neutralinos, viewed as the lightest supersymmetric particles (LSP). Such Cold Dark Matter candidates lead to a rich phenomenology of supersymmetric particle physics models. In the context of conventional Cosmology [16-19], some of these models can be constrained significantly by the recently available astrophysical data on Cosmic Microwave Background temperature fluctuations [15]. The calculation of relic abundances will be done in some detail, in order for the reader to appreciate better the rôle of the non conventional terms in (2.25).

It is convenient to write the Boltzmann equation for the density of species n in a compact form that represents collectively the dilaton-dissipative-source and non-critical-string contributions as external-source $\Gamma(t)n$ terms:

$$\frac{dn}{dt} + 3\frac{\dot{a}}{a}n = \Gamma(t)n + \int \frac{d^3p}{E} C[f], \quad \Gamma(t) \equiv \dot{\Phi} + \frac{1}{2}\eta \left(e^{-\Phi} g^{\mu\nu} \tilde{\beta}_{\mu\nu}^{\text{Grav}} + 2e^{\Phi} \tilde{\beta}^{\Phi} \right) \qquad (3.1)$$

where we work in the physical scheme (2.6) from now on, for which η = -1. Depending on the sign of $\Gamma(t)$ one has different effects on the relic abundance of the species X with density n, which we now proceed to analyze. To find an explicit expression for $\Gamma(t)$ in our case we should substitute the solution of (2.9), more specifically (2.11), analyzed in [1]. Regarding the form of (3.1)

it is nice to see that the extra terms can be cast in a simple-looking form of a source term $\Gamma(t)n$ including both the dilaton dissipation and the non-critical-string terms. Of course $\Gamma(t)$ is complicated and requires the full solution outlined in [1].

In a more familiar form, the interaction term $C[f]$ of the above modified Boltzmann equation can be expressed in terms of the thermal average of the cross section σ times the Moeller velocity v of the annihilated particles [21]

$$\frac{dn}{dt} = -3\frac{\dot{a}}{a}n - \langle v\sigma \rangle \; (n^2 - n_{eq}^2) + \Gamma n \tag{3.2}$$

Before the decoupling time t_f, $t < t_f$, equilibrium is maintained and thus $n = n_{eq}$ for such an era. However, it is crucial to observe that, as a result of the presence of the source Γ terms, n_{eq} no longer scales with the inverse of the cubic power of the expansion radius a, which was the case in conventional (on-shell) cosmological models.

To understand this, let us assume that $n = n_{eq}^{(0)}$ at a very early epoch t_0. Then the solution of the modified Boltzmann equations at all times $t < t_f$ is given by

$$n_{eq}a^3 = n_{eq}^{(0)} \; a^3(t_0) \; \exp\left(\int_{t_0}^{t} \Gamma dt\right). \tag{3.3}$$

The time t_0 characterizes a very early time, which is not unreasonable to assume that it signals the exit from the inflationary period. Soon after the exit from inflation, all particles are in thermal equilibrium, for all times $t < t_f$, with the source term modifying the usual Boltzmann distributions in the way indicated in Eq. (3.3) above. It has been tacitly assumed that the entropy is conserved despite the presence of the source and the non-critical-string contributions. In our approach this is an *approximation*, since we know that non-critical strings lead to entropy production. However, as argued in our previous works on the subject [2,3], the entropy increase is most significant during the inflationary era, and hence it is not inconsistent to assume that, for all practical purposes, sufficient for our phenomenological analysis in this work, there is no significant entropy production after the exit from inflation. This is a necessary ingredient for our approach, since without such an assumption no predictions can be made, even in the conventional cosmological scenarios. Thus, the picture we envisage is that at t_0 the Universe entered an equilibrium phase, the entropy is conserved to a good approximation, and hence all particle species find themselves in thermal equilibrium, despite the presence of the Γ source, which *slowly* pumps in or sucks out energy, without, however, disturbing the particles' thermal equilibrium.

From the above discussion it becomes evident that it is of paramount importance to know the behaviour of the source term at all times, in order to extract information for the relic abundances, especially those concerning Dark Matter, and how these are modified from those of the standard

Cosmology. Before embarking on such an enterprize and study the phenomenological conse-quences of particular models predicting the existence of Dark Matter, especially Supersymmetry-based ones, we must first proceed in a general way to set up the stage and discuss how the relic density is affected by the presence of the non conventional source terms discussed above.

For the sake of brevity, we shall not deploy all the details of the derivation of the relic density, but instead demonstrate the most important features and results of our approach, paying partic-ular attention to exhibiting the differences from the conventional case. Generalizing the standard techniques [21], we assume that above the freeze-out point the density is the equilibrium density as provided by Eq. (3.3), while below this the interaction terms starts becoming unimportant. It is customary to define $x \equiv T/m_{\tilde{\chi}}$ and restrict the discussion on a particular species $\tilde{\chi}$ of mass $m_{\tilde{\chi}}$, which eventually may play the role of the dominant Dark Matter candidate. It also proves con-venient to trade the number density n for the quantity $Y \equiv n/s$, that is the number per entropy density [21]. The equation for Y is derived from (3.2) and is given by

$$\frac{dY}{dx} = m_{\tilde{\chi}} \langle v\sigma \rangle \, (\frac{45}{\pi} G_N \tilde{g}_{eff})^{-1/2} \, (h + \frac{x}{3}\frac{dh}{dx})(Y^2 - Y_{eq}^2) - \frac{\Gamma}{Hx}(1 + \frac{x}{3h}\frac{dh}{dx})Y. \qquad (3.4)$$

where $G_N = 1/M_{Planck}^2$ is the four-dimensional gravitational constant, the quantity H is the Hubble expansion rate, h denote the entropy degrees of freedom, and $\langle v\sigma \rangle$ is the thermal average of the relative velocity times the annihilation cross section and $\tilde{g}_{eff}$ is simply defined by the rela-tion [21]

$$\varrho + \Delta\varrho \equiv \frac{\pi^2}{30} T^4 \tilde{g}_{eff}. \qquad (3.5)$$

The reader should notice at this point that $\Delta\varrho$ incorporates the effects of the additional contri-butions due to the non-critical (off-shell) terms and the dilaton dissipative source, which are not accounted for in the g_{eff} of conventional Cosmology [21], hence the notation $\tilde{g}_{eff}$. We next remark that ρ, as well as $\Delta\rho$, as functions of time are known, once one solves the cosmological equations. However, only the degrees of freedom involved in ρ are thermal, the rest, like the comological-constant term if present in a model, are included in $\Delta\rho$. Therefore, the relation between temperature and time is provided by

$$\rho = \frac{\pi^2}{30} T^4 g_{eff}(T) \qquad (3.6)$$

while $\rho + \Delta\rho$ are involved in the evolution through (c.f. (2.11))

$$H^2 = \frac{8\pi G_N}{3}(\rho + \Delta\rho). \tag{3.7}$$

Thus, it is important for the reader to bear in mind that $\Delta\rho$ contributes to the dynamical expansion, through Eq. (3.7), but not to the thermal evolution of the Universe. The quantity $\tilde{g}_{eff}$, defined in (3.5), is therefore given by

$$\tilde{g}_{eff} = g_{eff} + \frac{30}{\pi^2} T^{-4} \Delta\rho. \tag{3.8}$$

The meaning of the above expression is that time has been replaced by temperature, through Eq. (3.6), after solving the dynamical equations. In terms of $\tilde{g}_{eff}$ the expansion rate H is written as

$$H^2 = \frac{4\pi^3 G_N}{45} T^4 \tilde{g}_{eff}. \tag{3.9}$$

This is used in the Boltzmann equation for Y and the conversion from the time variable t to temperature or, equivalently, the variable x.

For x above the freezing point x_f, $Y \approx Y_{eq}$ and, upon omitting the contributions of the derivative terms dh/dx, an approximation which is also adopted in the standard cosmological treatments [21], we obtain for the solution of (3.4)

$$Y_{eq} = Y_{eq}^{(0)} \exp(-\int_x^\infty \frac{\Gamma H^{-1}}{x} dx). \tag{3.10}$$

Here, $Y_{eq}^{(0)}$ corresponds to $n_{eq}^{(0)}$ and in the non-relativistic limit is given by

$$Y_{eq}^{(0)} = \frac{45}{2\pi^2} \frac{g_s}{h} (2\pi x)^{-3/2} \exp(-1/x) \tag{3.11}$$

where g_s counts the particle's spin degrees of freedom.

In the regime $x < x_f$, $Y >> Y_{eq}^{(0)}$ the equation (3.4) can be written as

$$\frac{d}{dx}\frac{1}{Y} = -m_{\tilde{\chi}}\langle v\sigma\rangle\,(\frac{45}{\pi}G_N\tilde{g}_{eff})^{-\frac{1}{2}}h + \frac{\Gamma H^{-1}}{xY} \qquad (3.12)$$

Applying (3.12) at the freezing point x_f and using (3.10) and (3.11), leads, after a straightforward calculation, to the determination of $x_f = T_f/m_{\tilde{\chi}}$ through

$$x_f^{-1} = ln\left[\, 0.03824\, g_s\, \frac{M_{Planck}m_{\tilde{\chi}}}{\sqrt{g_*}}\, x_f^{1/2}\langle v\sigma\rangle_f\,\right] + \frac{1}{2}ln\left(\frac{g_*}{\tilde{g}_*}\right) + \int_{x_f}^{x_{in}}\frac{\Gamma H^{-1}}{x}dx. \qquad (3.13)$$

As usual, all quantities are expressed in terms of the dimensionless $x \equiv T/m_{\tilde{\chi}}$ and x_{in} corresponds to the time t_0 discussed previously, taken to represent the exit from the inflationary period of the Universe.

The first term on the right-hand-side of (3.13) is that of a conventional Cosmology for, say, an LSP carrying g_s spin degrees of freedom, playing the rôle of the dominant Cold Dark Matter species in a concrete and physically promising example [16-19], which we use in this work. The quantity $\langle v\sigma\rangle_f$ is the thermal average of $v\sigma$ at x_f and g_* is g_{eff} of conventional Cosmology at the freeze-out point. The same notation holds for $\tilde{g}_*$. In our treatment above, we chose in (3.13) to present x_f in such a way so as to separate the conventional contributions, which reside in the first term, from the contributions of the dilaton and the non-critical-string dynamics, which are contained within the last two terms. The latter induce a shift in the freeze-out temperature. The penultimate term on the right hand side of (3.13), due to its logarithmic nature, does not affect much the freeze-out temperature. The last term, on the other hand, is more important and, depending on its sign, may shift the freeze-out point to earlier or later times. To quantify the amount of the shift one must solve the equations (2.11) of ref. [1]. We shall do this in a forthcoming publication [28], where we shall also present a detailed analysis of the effects of the non-critical string and dilaton-source terms on the constraints on supersymmetric particle physics models, extending conventional cosmology works [16-19].

For our purposes here we note that, in order to calculate the relic abundance, we must solve (3.12) from x_f to today's value x_0, corresponding to a temperature $T_0 \approx 2.7\,°K$. Following the usual approximations we arrive at the result:

$$Y^{-1}(x_0) = Y^{-1}(x_f) + (\frac{\pi}{45})^{\frac{1}{2}}m_{\tilde{\chi}}M_{Planck}\tilde{g}_*^{-\frac{1}{2}}h(x_0)J - \int_{x_0}^{x_f}\frac{\Gamma H^{-1}}{xY}dx. \qquad (3.14)$$

In conventional Cosmology [21] $\tilde{g}_*$ is replaced by g_* and the last term in (3.14) is absent. The quantity J is $J \equiv \int_{x_0}^{x_f} \langle v\sigma \rangle dx$. By replacing $Y(x_f)$ by its equilibrium value (3.10) the ratio of the first term on the r.h.s. of (3.14) to the second is found to be exactly the same as in the no-dilaton case. Therefore, by the same token as in conventional Cosmology, the first term can be safely omitted, as long as x_f is of order of 1/10 or less. Furthermore, the integral on the r.h.s. of (3.14) can be simplified if one uses the fact that $\langle v\sigma \rangle n$ is small as compared with the expansion rate $\dot{a}/a$ after decoupling. For the purposes of the evaluation of this integral, therefore, this term can be omitted in (3.12), as long as we stay within the decoupling regime, and one obtains:

$$\frac{d}{dx}\frac{1}{Y} = \frac{\Gamma H^{-1}}{xY}. \qquad (3.15)$$

By integration this yields $Y(x) = Y(x_0) \, \exp(-\int_{x_0}^{x} \Gamma H^{-1} dx/x)$. Using this inside the integral in (3.14) we get

$$(h(x_0)Y(x_0))^{-1} = \left(1 + \int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{\psi(x)} dx \right)^{-1} (\frac{\pi}{45})^{\frac{1}{2}} m_{\tilde{\chi}} M_{Planck} \tilde{g}_*^{-\frac{1}{2}} J \qquad (3.16)$$

where the function $\psi(x)$ is given by $\psi(x) \equiv x\exp(-\int_{x_0}^{x} \Gamma H^{-1} dx/x)$. With the exception of the prefactor on the r.h.s. of (3.16), this is identical in form to the result derived in standard treatments, if $\tilde{g}_*$ is replaced by g_* and the value of x_f, implicitly involved in the integral J, is replaced by its value found in ordinary treatments in which the dilaton-dynamics and non-critical-string effects are absent.

The matter density of species $\tilde{\chi}$ is then given by

$$\rho_{\tilde{\chi}} = f \left(\frac{4\pi^3}{45} \right)^{1/2} \left(\frac{T_{\tilde{\chi}}}{T_\gamma} \right)^3 \frac{T_\gamma^3}{M_{Planck}} \frac{\sqrt{\tilde{g}_*}}{J} \qquad (3.17)$$

where the prefactor f is:

$$f = 1 + \int_{x_0}^{x_f} \frac{\Gamma H^{-1}}{\psi(x)}$$

It is important to recall that the thermal degrees of freedom are counted by g_{eff}(c.f. (3.6)), and not $\tilde{g}_{eff}$, the latter being merely a convenient device connecting the total energy, thermal and non-thermal, to the temperature T (c.f. (3.5)). Hence,

$$\left(\frac{T_{\tilde{\chi}}}{T_{\gamma}}\right)^3 = \frac{g_{eff}(1MeV)}{g_{eff}(T_{\tilde{\chi}})}\frac{4}{11} = \frac{43}{11}\frac{1}{g_*}. \tag{3.18}$$

In deriving (3.18) only the thermal content of the Universe is used, while the dilaton and the non-critical terms do not participate. Therefore the $\tilde{\chi}$'s matter density is given by

$$\rho_{\tilde{\chi}} = f\left(\frac{4\pi^3}{45}\right)^{1/2}\frac{43}{11}\frac{T_{\gamma}^3}{M_{Planck}}\frac{\sqrt{\tilde{g}}_*}{g_* J}. \tag{3.19}$$

This formula tacitly assumes that the $\tilde{\chi}$s decoupled before neutrinos. For the relic abundance, then, we derive the following approximate result

$$\Omega_{\tilde{\chi}}h_0^2 = \left(\Omega_{\tilde{\chi}}h_0^2\right)_{no-source} \times \left(\frac{\tilde{g}_*}{g_*}\right)^{1/2} \exp\left(\int_{x_0}^{x_f}\frac{\Gamma H^{-1}}{x}dx\right), \tag{3.20}$$

where we used the fact that: $1 + \int_{x_0}^{x_f}\frac{\Gamma H^{-1}}{\psi(x)}dx = \exp\left(\int_{x_0}^{x_f}\frac{\Gamma H^{-1}}{x}dx\right)$. In (3.20), the quantity referred to as *no-source* is the well known no-source expression

$$\left(\Omega_{\tilde{\chi}}h_0^2\right)_{no-source} = \frac{1.066\times10^9 GeV^{-1}}{M_{Planck}\sqrt{g_*}\ J} \tag{3.21}$$

where $J \equiv \int_{x_0}^{x_f}\langle v\sigma\rangle dx$. However, as already remarked, the end point x_f in the integration is the shifted freeze-out point as determined by Eq. (3.13). The merit of casting the relic density in such a form is that it clearly exhibits the effect of the presence of the source. Certainly, if an accurate result is required, one can proceed without approximations and handle the problem numerically as in the standard treatments. We shall present such a more complete analysis in a forthcoming publication [28].

4 Discussion: Source Effects on Particle Phenomenology
From the expression (3.20) above, it becomes evident that the effects of the integral involving the source Γ on the relic density evolution may be quite important. Indeed, if Γ is kept negative at all times, this results in reduction of the relic density with time, contrary to what happens in the case

where Γ is positive. In the former case, predictions for supersymmetric models [16-19] can be drastically altered, since the parameter space is enlarged, leaving more room for supersymmetry, probably beyond the reach of LHC, even in the case of constrained minimal supersymmetric standard models with compact parameter spaces of the embedding minimal supergravity theory. The opposite happens in the case of positive Γ, where the parameter space is shrunk and predictions can be very restrictive to almost excluding supersymmetry, especially if the prefactor turns out to be a large number.

In order to get a rough picture of the importance of such changes in the calculation of the relic density, let us assume that $\dot{a}/a + \dot{\Phi} = 0$, as would be the case in the present era if matter were absent [25]. The non-critical terms would also contribute little in this case and from (3.1) one would have $\Gamma \approx \dot{\Phi}$. Then, as follows from (3.1), the density of the dark matter species n would scale (with the scale factor) like radiation, a^{-4} (which in our approach retains its conventional cosmology scaling behaviour, as a result of the coupling of the gauge fields to the dilaton in the low-energy string inspired effective (slightly off-shell) action, which assumes the standard form). Therefore, $\Gamma H^{-1} \approx -1$ and the function $\psi(x)$ would be $\psi(x) \approx x^2/x_0$. As a result, the prefactor in (3.20) becomes x_0/x_f which is an enormously small number, of order $\sim 10^{-13}$ or so. Such a small number would result practically to no cosmological constraints on supersymmetric models! However, if this were the case, an even more dramatic situation would have taken place: the above unconventional scaling of dark matter, which is the dominant species, could not be disentangled from radiation, and the universe would never become matter dominated, in the standard cosmology sense, thus affecting significantly structure formation.

However this situation is not realized in nature, since at the present era neither matter nor the non-critical term contributions are negligible [1]. A decent approach is to solve the cosmological equations (2.11) and thus obtain the function $\Gamma(t)$ at all times, t, from today to the remote past. This would allow for precise information to be obtained on the value of the freezing point x_f and of the prefactor appearing in the relic density (3.20). Thus, a complete numerical treatment, along the lines presented in [1], is needed in order to tackle important phenomenological questions regarding the relic abundance of Dark Matter in this framework. Nevertheless, the smallness of the prefactor in the crude approximation we have employed above, gives us a sneak preview of the drastic changes one may be faced with, as a result of the non-critical string dynamics.

A detailed phenomenology programme along these lines has started becoming available in [28], where the effects of matter in the above framework have been properly taken into account. In what follows we shall describe briefly the most important of these results for completeness. For details we refer the reader to [28]. When ordinary matter ("dust"), with scaling a^{-3} is present, then, solving the dynamical equations (2.11), as in [1], results in a complicated time dependence

of the dilaton field, which is such that $\dot{\Phi} + \dot{a}/a \neq 0$. This affects the scaling of the dark matter species, in such a way that, although dark matter components with "exotic" scaling are present in past eras, nevertheless the density of the latter can be made negligible during the present epoch and is such that the nucleosynthesis (occurring at MeV scales as usual) is also not affected. In such scenarios [28], the neutralino dark matter density can be diluted only by a factor of order $O(10)$, while the baryon energy density is diluted only by a factor of order $O(1)$, upon adjusting only one parameter, namely the equation of state of the neutralino [28], assuming it to be the dominant dark matter species. This implies that dark matter enhances its odds, and future colliders, such as the LHC, have more available supersymmetry parameter space to exploit!

It must be noted at this stage, however, that the above conclusions are rather phenomenological, in the sense of being derived by exploiting the effective dynamical equations of the low energy four-dimensional world, assuming generic matter and radiation densities, in the spirit of standard cosmology analyses. In realistic string models, the results depend crucially on many details of the underlying microscopic model, such as the compactification procedure, the details of supersymmetry breaking, the geometry of the bulk space in brane models, string loop corrections *etc*. Regarding the latter effects, we also mention that at late eras of the universe, the various scaling components of the energy density, such as a^{-4}, a^{-3}, *etc*, involve mixed contributions from *both* matter and gravitational/dilaton and radiation parts, in such a way that one may even have negative dust contributions during the present era in some models [20], e.g. as a result of Kaluza-Klein graviton modes in some brane cosmology scenarios. Such unconventional energy budget can still be made compatible at least with supernovae data [20]. It remains to be seen of course what constraints the CMB and structure formation data imply on such cosmologies, but it is evident from the above that string cosmology, and in particular supercritical string one, may lead to fascinating new results which may hide further surprises around the corner.

Moreover, in such a framework one could also tackle other important issues, such as a possible resolution of the gravitino overproduction problem in effective supergravity inflationary models [29-31]. In the non-critical string framework discussed above, the gravitino, as a member of the gravitational supermultiplet of the string, will feel the non-critical and dilaton dissipation terms, while matter effects are unimportant for this issue. Thus, its density may be substantially reduced, in a way similar to that discussed above, as compared with conventional supergravity scenarios [28]. *affair à suivre...*

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